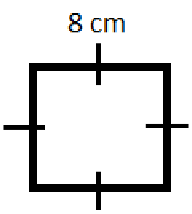
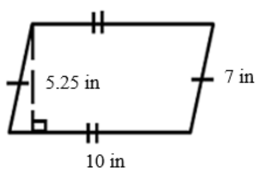
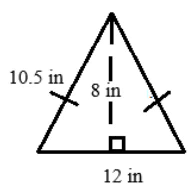
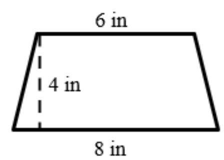
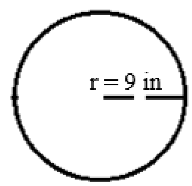
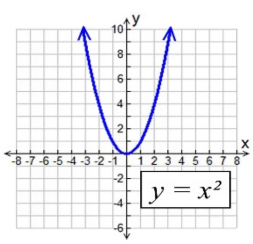
Name: ____________________
Teacher: __________________
Subject/Period ________________
Date: ____________________

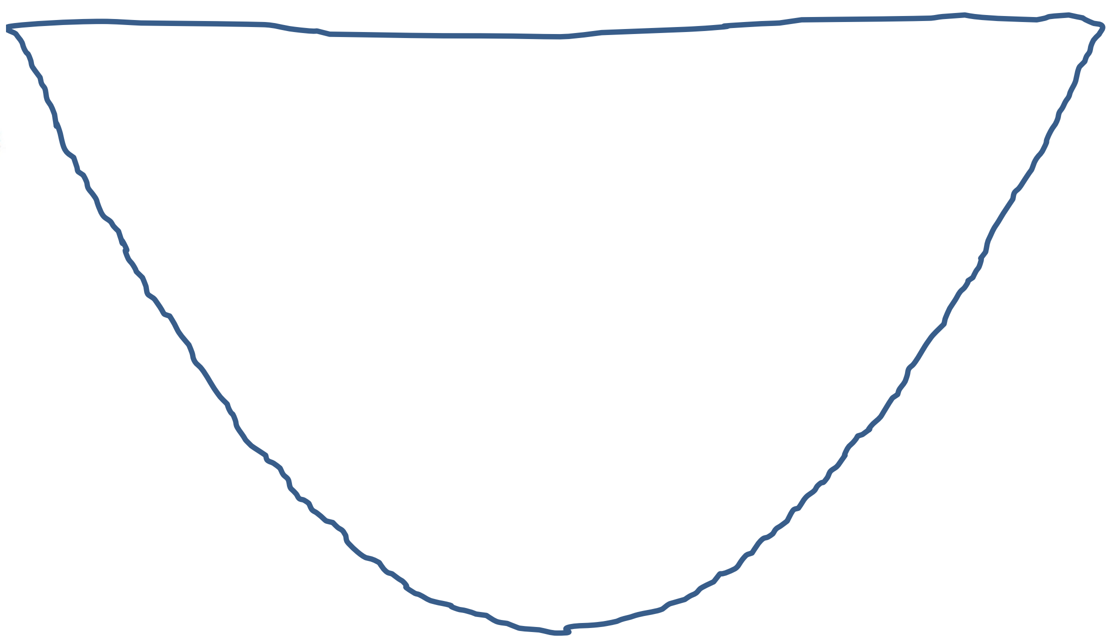
Calculus (Integration) for 8th Grade Math & Algebra I Students

Engage: Use the area formulas to calculate the area of these 2D shapes.
Use your calculator, but substitute values into the formulas first to show your work!

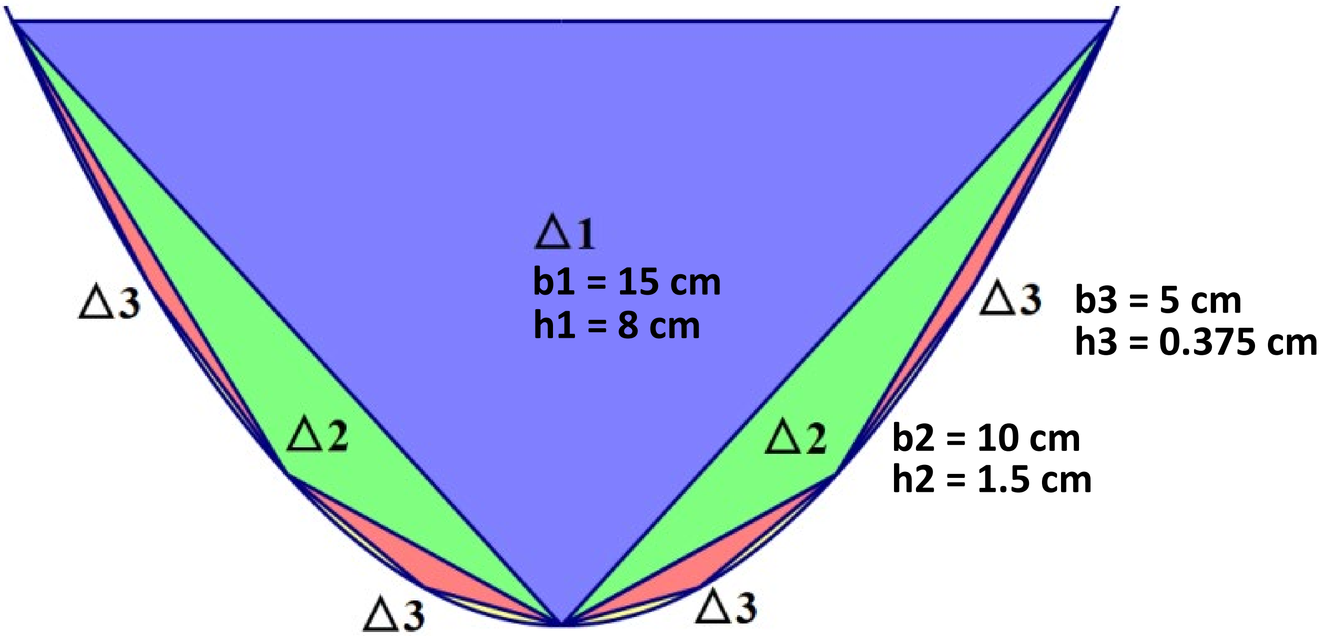
Rectangle	Parallelogram	Triangle	Trapezoid	Circle
$A = lw$	$A = bh$	$A = \frac{bh}{2}$	$A = \frac{(b_1 + b_2)h}{2}$	$A = \pi r^2$
				

Explore: How can we approximate the area inside this "parabola" using our skills for calculating the area of 2D shapes? (Note: when measuring your shapes, use centimeters.)



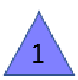


Explain: How did Archimedes calculate the area inside this parabola?

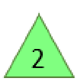


Calculate the area inside the parabola using Archimedes' triangles.
Note: You do not need to measure the dimensions yourself.

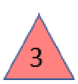
Step 1: Calculate the area of each triangle.



$$\text{Purple area} = \frac{bh}{2} =$$



$$\text{Green area} = \frac{bh}{2} =$$



$$\text{Red area} = \frac{bh}{2} =$$

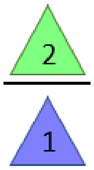
Step 2: Fill in the table and do some calculations to approximate the area inside the parabola.

Areas of Triangles	x num of triangles	Area covered inside parabola
Purple triangle area: _____	x 1 =	_____
Green triangle area: _____	x 2 =	_____
Red triangle area: _____	x 4 =	_____
Approximate area inside parabola:		_____

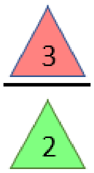
Class Discussion: Answer the questions below and be prepared to discuss what you think.

- Why isn't our answer accurate?
- What could we do to make our answer more accurate?
- Why is it difficult to get an accurate answer?

Archimedes found an accurate answer.
How did he do it? He noticed a pattern.
Can you find it? (Hint: simplify the fractions or convert them to decimals.)



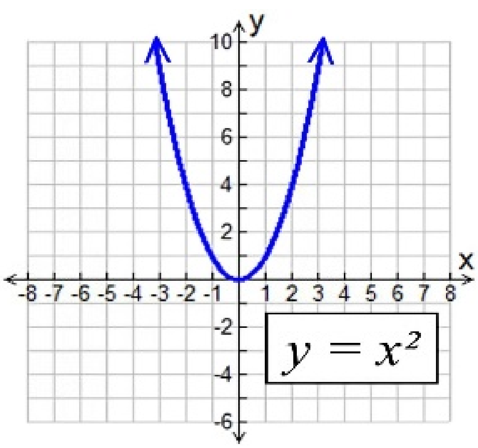
$$= \frac{Area\ of\ green\ triangle}{Area\ of\ purple\ triangle} = \frac{7.5}{60}$$

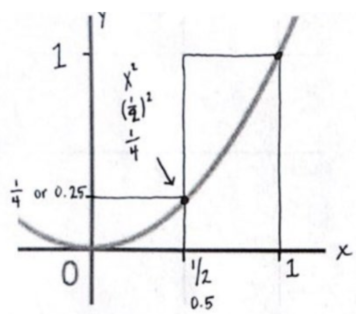


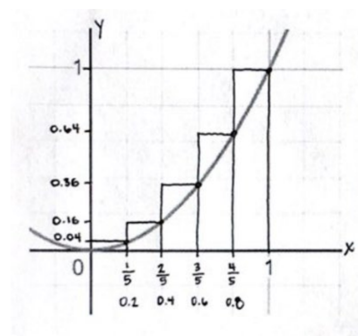
$$= \frac{Area\ of\ red\ triangle}{Area\ of\ green\ triangle} = \frac{0.9375}{7.5}$$

HOW DOES THIS RELATE TO CALCULUS?

In calculus, we use ________________ to calculate the area inside (or "under") curved shapes.







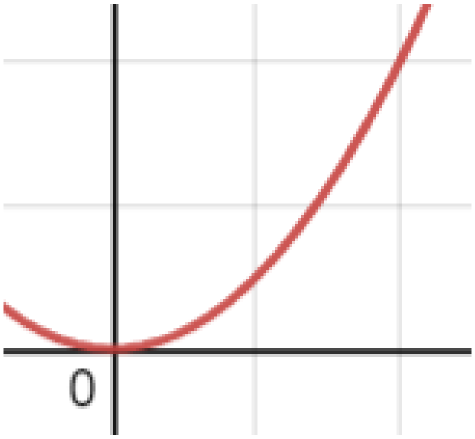
$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Bigg|_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3} - 0 = \boxed{\frac{1}{3}\ or\ 0.333\ldots\ square\ units}$$

Elaborate: In addition to Archimedes "triangle method," there are other ways to approximate the area inside shapes with curves. Let's learn about Sir Isaac Newton's "rectangle method."

Example 1: Integration for $y = x^2$ from x = 0 to x = 1.

$$\int_0^1 x^2\,dx = \frac{x^3}{3} = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3} - 0 = \boxed{\frac{1}{3}\, or\ 0.333 \ldots\, square\ units}$$

Accurate area under the curve

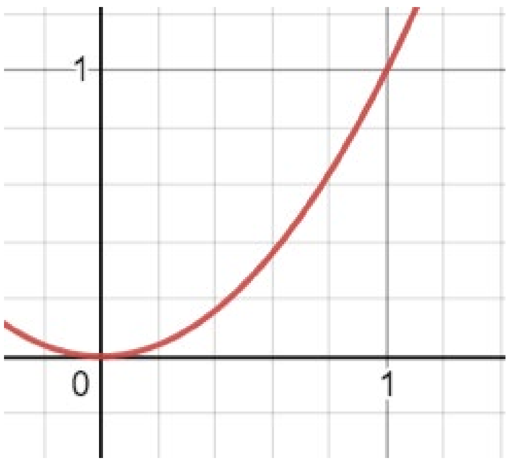


1. What is the length of the base of each rectangle?
   _________ (<- in calculus, this is called a partition)

2. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (_____ * _____) + (_____ * _____)
   A = (___________) + (___________)
   A = ______________________________ square units

3. Is the area approximation greater or less than 1/3 or 0.333…? (Circle one.)
   Greater than 1/3 or 0.333…   Less than 1/3 or 0.333…

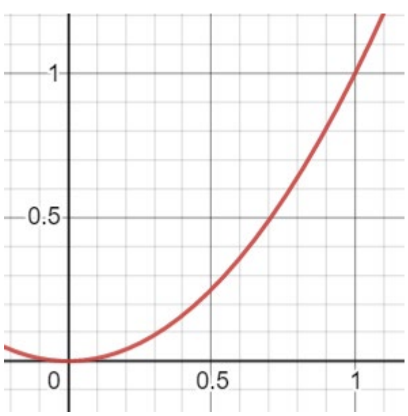


4. What is the length of the base of each rectangle?
   _________ (<- in calculus, this is called a partition)

5. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (__ * __) + (__ * __) + (__ * __) + (__ * __) + (__ * __)
   A = (_______) + (_______) + (_______) + (_______) + (_______)
   A = ______________________________ square units

6. Is the area approximation greater or less than 1/3 or 0.333…? (Circle one.)
   Greater than 1/3 or 0.333…   Less than 1/3 or 0.333…



7. What is the length of the base of each rectangle?
   _________ (<- in calculus, this is called a partition)

8. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (__ * __) + (__ * __) + (__ * __) + (__ * __) + (__ * __)
   + (__ * __) + (__ * __) + (__ * __) + (__ * __) + (__ * __)
   A = (__)+(__)+(__)+(__)+(__)+(__)+(__)+(__)+(__)+(__)
   A = ______________________________ square units

9. Is the area approximation greater or less than 1/3 or 0.333…? (Circle one.)
   Greater than 1/3 or 0.333…   Less than 1/3 or 0.333…

Answer these questions…

10. Which rectangle method's approximate area under the curve was closest to the accurate area? (Circle one.)

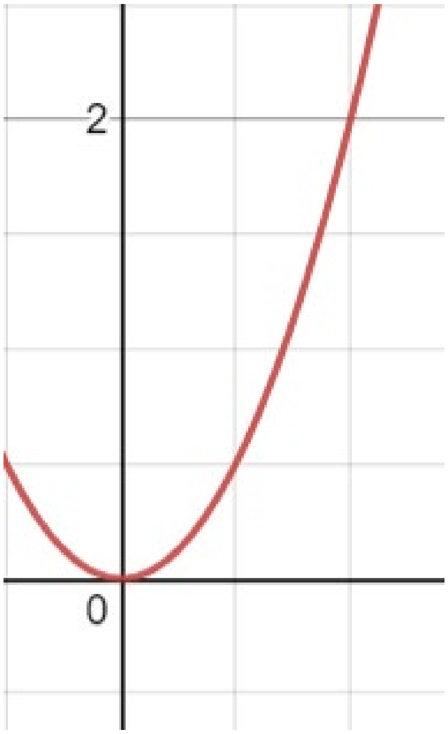
    Rectangles with a partition of ½   Rectangles with a partition of 1/5   Rectangles with a partition of 1/10

11. All of the area approximations are greater than the accurate answer (1/3 or 0.333…). Why do you think that is?

Example 2: Integration for $y = 2x^2$ from x = 0 to x = 1.

$$\int_0^1 2x^2\,dx = \frac{2x^3}{3} = \frac{2(1^3)}{3} - \frac{2(0^3)}{3} = \frac{2}{3} - 0 = \boxed{\frac{2}{3}\, or\ 0.666\ldots square\ units}$$

Accurate area under the curve

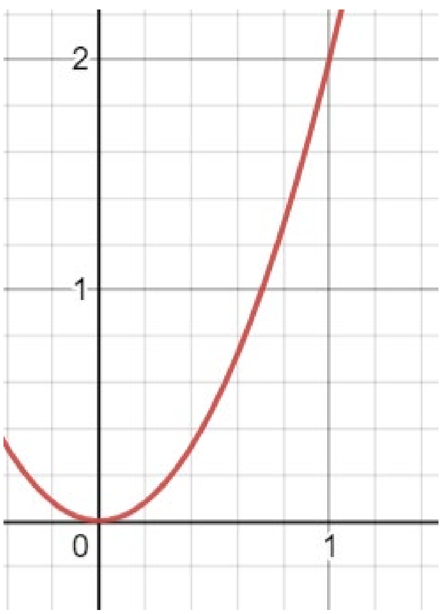


1. What is the length of the base of each rectangle?
   _________ (<- in calculus, this is called a partition)

2. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (_____ * _____) + (_____ * _____)
   A = (___________) + (___________)
   A = ______________________________ square units

3. Is the area approximation greater or less than 2/3 or 0.666…? (Circle one.)
   Greater than 2/3 or 0.666…   Less than 2/3 or 0.666…



4. What is the length of the base of each rectangle?
   _________ (<- in calculus, this is called a partition)

5. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (__ * __) + (__ * __) + (__ * __) + (__ * __) + (__ * __)
   A = (_______) + (_______) + (_______) + (_______) + (_______)
   A = ______________________________ square units

6. Is the area approximation greater or less than 2/3 or 0.666…? (Circle one.)
   Greater than 2/3 or 0.666…   Less than 2/3 or 0.666…

Answer these questions…

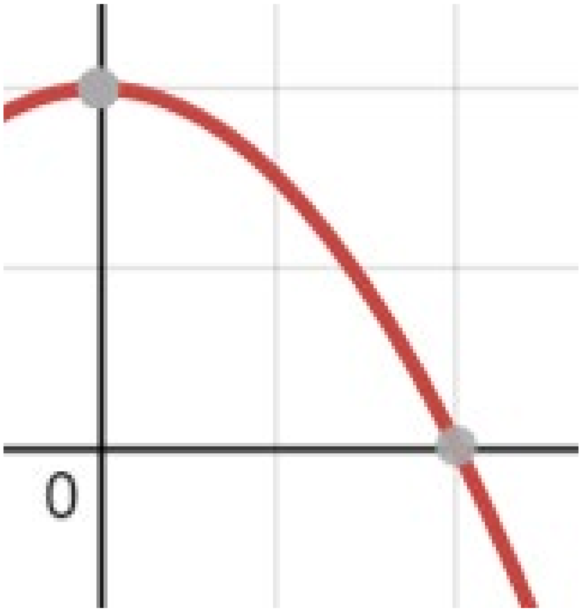
7. How does the size of the partition change the accuracy of the approximation of the area under the curve? Why?

8. What similarities and differences do you notice between the “triangle method” and the “rectangle method”?

Evaluate: Complete Example 3 on your own. You can do it!

Example 3: Integration for $y = -x^2 + 1$ from x = 0 to x = 1.

$$\int_0^1 -x^2 + 1\,dx \;=\; \frac{-x^3}{3} + x \;=\; \left(\frac{-(1^3)}{3} + 1\right) - \left(\frac{-(0^3)}{3} + 0\right) \;=\; \frac{-1}{3} + 1 \;=\; \boxed{\frac{2}{3}\, or\ 0.666\ldots square\ units}$$

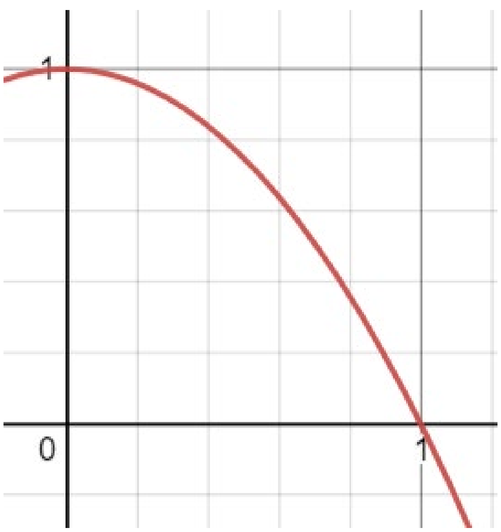


1. What is the length of the base of each rectangle?
   _________ (<- in calculus, this is called a partition)

2. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (____ * ____) + (____ * ____)
   A = (___________) + (___________)
   A = _____________________________ square units

3. Notice that the approximation is less than 2/3 or 0.666…. Why do you think this approximation is less than the accurate answer?



4. What is the length of the base of each rectangle?
   _________ (<- in calculus, this is called a partition)

5. Calculate the area of each rectangle, then sum the areas to approximate the area under the curve.

   A = (__ * __) + (__ * __) + (__ * __) + (__ * __) + (__ * __)
   A = (______) + (______) + (______) + (______) + (______)
   A = _____________________________ square units

6. Which approximation (the approximation above with a partition of ½ or this approximation with a partition of 1/5) is closer to the accurate answer? Why?

Answer this question…

7. What did you learn today in your own words? Give as much detail as you can.