The Impacts of John von Neumann's Multidisciplinary Mind

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Theory of Calculus

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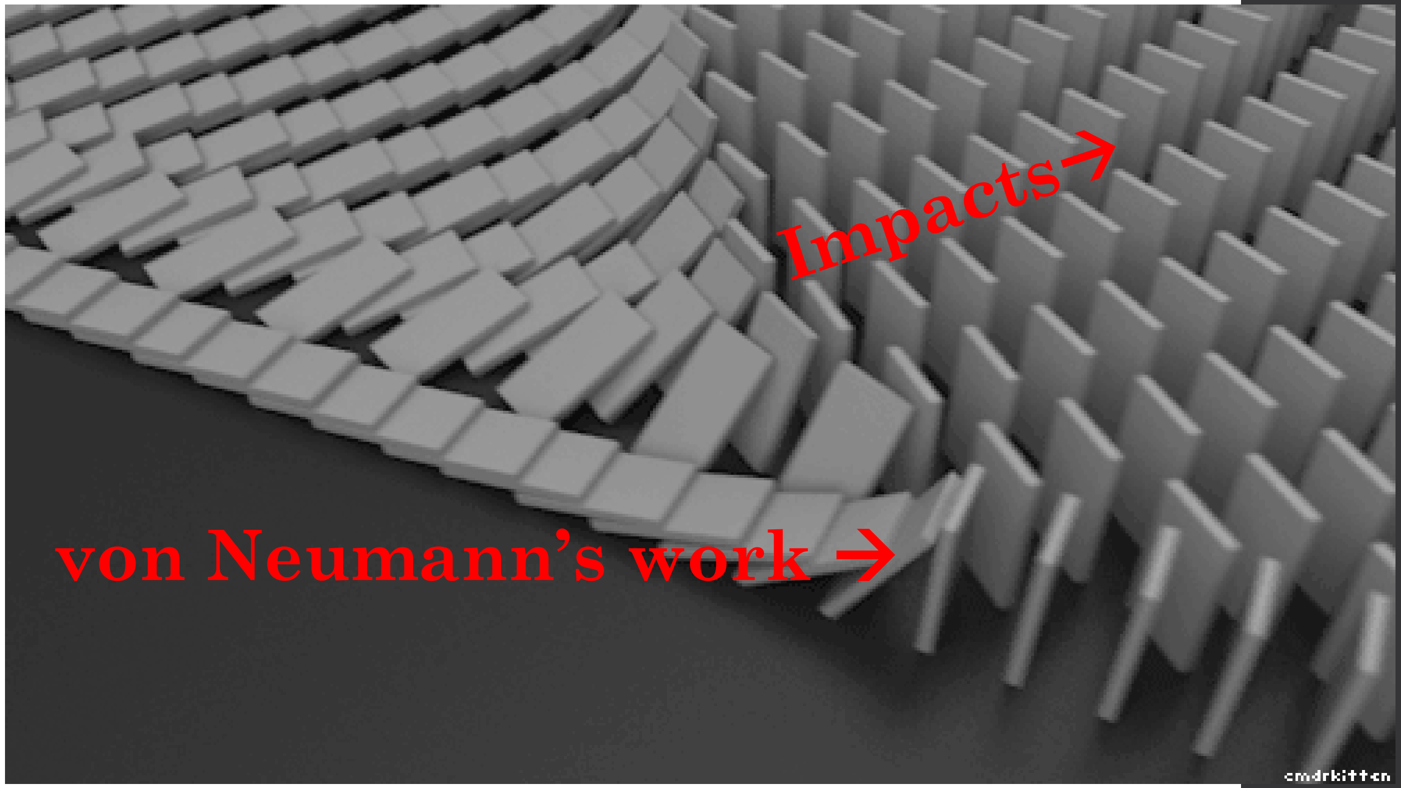
John Von Neumann's Life (Veisdal, 2019)

- Born in 1903 in Budapest, Hungary to a wealthy family
- Displayed early prodigious talents in mathematics and had an eidetic memory
- Earned a degree in chemical engineering and a PhD in mathematics in 1926 at age 24
- Worked with many significantly accomplished people over the course of his life:
  - Eugene Wigner –atomic nuclear theory & quantum mechanics
  - Hermann Weyl – Hilbert's theory of consistency
  - Albert Einstein – general theory of relativity
  - David Hilbert – multi-dimensional mathematics
  - Alan Turing – computing and code breaking
  - Oskar Morgenstern –economic game theory
  - Niels Bohr – atomic structure and quantum mechanics
  - Richard Feynman – quantum mechanics
  - J. Robert Oppenheimer – nuclear weapons
- Spoke Hungarian, German, English, French, and Italian
- Became a visiting professor at Princeton in 1930 and immigrated to the US in 1933
- Offered a lifetime professorship at Massachusetts Institute of Advanced Study in 1933
- Died in 1957 at the age of 53 after diagnoses of pancreatic and prostate cancer

Impacts of von Neumann’s Work

“Stanislaw Ulam, one of von Neumann’s close friends, described von Neumann’s mastery of mathematics as follows:

‘Most mathematicians know one method… **John von Neumann mastered three methods:** 1) A facility for the symbolic manipulation of linear operators, 2) an intuitive feeling for the logical structure of any new mathematical theory; and 3) an intuitive feeling for the combinatorial superstructure of new theories’” (Veisdal, 2019).



Structure for this Presentation

What were John von Neumann's areas of impact? (Veisdal, 2019)

→

What were his motivations for pursuing work in those areas? (Macrae, 2016)

→

What is an example of an accomplish-ment in that area? (various sources)

Which area of impact would you like to learn more about? (click on an icon)

Mathematics

Physics

Statistics

Economics

Atomic Bomb

Digital Computing

Go to Bibliography

Mathematics Motivation

“[von Neumann] wanted in the 1920s to become the supreme mathematical logician, establishing in infinite dimensions (but preferably on a single sheet of paper) a modern version of the axioms by which the ancient Greeks and others had managed for twenty-three hundred years to make mathematics seem entirely rigorous. **‘Rigor’ means obeying rules that this calculation follows certainly from that one, which follows undeniably from that one, allowing mathematicians to follow a line of thought that is absolutely guaranteed, thus establishing mathematics’ claim to be the foundation of all rationality.** [He] wanted to return math to the role of leading all intellectual progress, instead of so much of it being ad hoc. **He wanted, by axiomatization, to rescue the then most modern part of math (set theory) from its alleged contradictions”** (Macre, 2016).

RIGOR

Mathematics Example

"In his fourth set theory paper, entitled *Die Axiomatisierung der Mengenlehre* ("The Axiomatization of Set Theory"), von Neumann formally lays out his own axiomatic system. With its single page of axioms, it was the most succinct set theory axioms developed at the time, and formed the basis for the system later developed by Gödel and Berneys" (Veisdal, 2019).

Neumann-Bernays-Gödel axioms	
(1)	*Axiom of extension.* If A and B are classes and if, for all (sets) x, $x \in A$ if and only if $x \in B$, then $A = B$.
(2)	*Axiom of the empty set.* There exists a set A such that, for all x, it is false that $x \in A$.
(3)	*Axiom schema for class formation.* If $S(x)$ is a condition on x in which only set variables are introduced by the phrase "for all" or "for some" and in which B is not free, then there exists a class B such that $x \in B$ if and only if $S(x)$.
(4)	*Axiom of pairing.* If A and B are sets, there exists a set (symbolized $\{A, B\}$ and called the unordered pair of A and B) having A and B as its sole members.
(5)	*Axiom of union.* If C is a set, there exists a set A such that $x \in A$ if and only if $x \in B$ for some member B of C.
(6)	*Axiom of power set.* If A is a set, there exists a set B, called its power set, such that $x \in B$ if and only if $x \subseteq A$.
(7)	*Axiom of infinity.* There exists a set A such that $\varnothing \in A$ and, if $x \in A$, then $(x \cup \{x\}) \in A$, in which $x \cup \{x\}$ is the set x with x adjoined as a further member.
(8)	*Axiom of choice.* If A is a set the elements of which are nonempty sets, then there exists a function f with domain A such that, for each member B of A, $f(B) \in B$.
(9)	*Axiom of replacement.* If (the class) X is a function and A is a set, then there exists a set B such that $y \in B$ if and only if, for some x, $(x, y) \in X$ and $x \in A$; i.e., the range of the restriction of a function X to a domain that is a set is also a set.
(10)	*Axiom of restriction (foundation axiom).* Every nonempty class A contains an element B such that $A \cap B = \varnothing$.

Physics Motivations

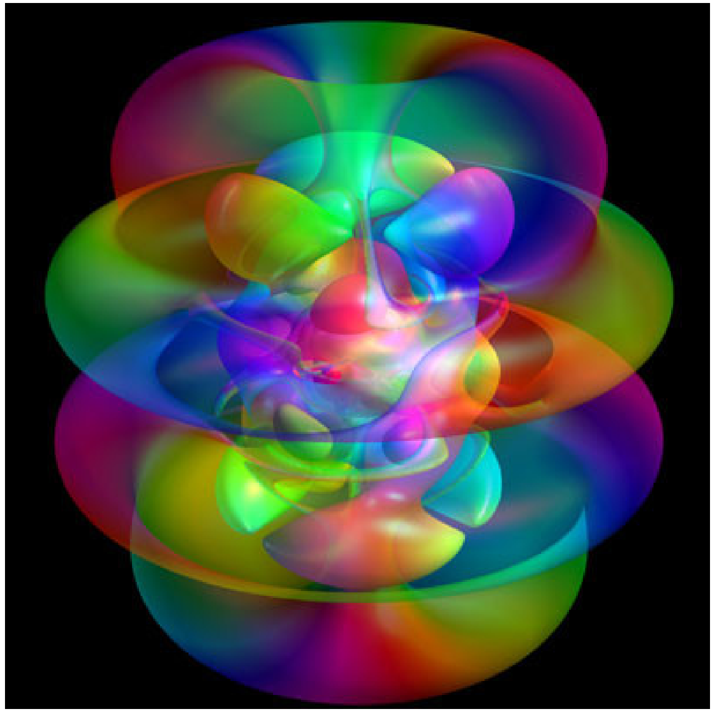
"Some people will feel that such an abstract space [one with more than three dimensions], largely unexplored by its inventor [Hilbert], is terribly theoretical and muddling. Others will find it enormously challenging fun. **[von Neumann] was the human being who found it the most challenging fun.** He spent much of his next dozen years probing intellectually into Hilbert space, producing (sometimes in collaboration with other fine minds like F. J. Murray's) nearly sixty papers and over a thousand startling ideas. **Each of these new ideas and discoveries was aesthetically pleasing to him because he was treading on ground that he thought rather few had reached before, and he was quick to think of useful applications for many of the things he found"** (Macrae, 2016).

FUN

Physics Example

“[von Neumann’s] basic insight, which neither Heisenberg, Bohr or Schrödinger had… was ‘that the geometry of the vectors in a Hilbert space have the same formal properties as the structure of the states of a quantum mechanical system’”(Veisdal, 2019).

“[von Neumann] found that what really determines the dimensional structure of a space is the group of rotations that it permits… he helped mathematicians not only to do sums in infinite dimensions but also to think (if they wished) in such weirdnesses as one-third of a dimension, or the square root of two as a dimension, or π as a dimension, or any other real number they wanted” (Macrae, 2016).



Statistics Motivation

“**Because [von Neumann] was a recognized expert on Hilbert space, other people’s wanderings in it were referred to him. When B. O. Koopman in 1931 showed how to formulate the ergodic theorem in terms of Hilbert space, [he] accepted this as ‘a challenge and a hint.’** The ergodic theorem was crucial for the foundation of statistical mechanics. When scientists in the nineteenth century sought to explain the behavior of liquids and gases on the basis of Newton’s laws of mechanics, they did so by taking averages. Johnny’s ‘naïve’ ergodic theorem was briefly celebrated as the first rigorous mathematical basis for this sort of statistical mechanics.

Within a very short time, G. D. Birkhoff from Harvard had greatly strengthened and improved on [von Neumann’s] ergodic theorem. Some people thought [he] was cross about this, especially because Birkhoff was able to dash into print faster… [von Neumann] expressed pleasure rather than resentment, although he did kick himself for not spotting the next steps from his own calculations that Birkhoff saw. **He also felt that the European system (where keen minds would gather together around blackboards to help forward each other’s ideas) worked better than the American system of claiming personal priority for everything”** (Macrae, 2016).

COLLABORATION

Statistics Example

“Ergodic theory is the branch of mathematics that studies the statistical properties of deterministic dynamical systems. Formally, ergodic theory is concerned with *the states of dynamical systems with an invariant measure… In* two papers published in 1932, von Neumann made foundational contributions to the theory of such systems, including the von Neumann’s mean ergodic theorem, considered the first rigorous mathematical basis for the statistical mechanics of liquids and gases” (Veisdal, 2019).

Theorem 1 (von Neumann’s ergodic Theorem) *Let* (X, μ, T) *be an ergodic m.p.s. and let* $f \in L^2(X)$*. Then*

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f(T^n x) = \int_X f \, d\mu \qquad \text{in } L^2(X)$$

Economics Motivation

“Morgenstern was once asked how a scholar outside the mainstream of economic thinking could make a contribution to economics as original, innovative, and decisive as [von Neumann’s] EEM. Morgenstern replied that **[von Neumann] had an extraordinary capacity to pick the brains of a person whom he engaged in casual conversations. Once he saw from these that there was a problem of sufficient mathematical interest to warrant his spending of time on it, he homed on to that subject like a guided missile…**

In economics this meant he turned to decision theory, and particularly to how a businessman should operate when seeking a maximum of profit. Even in 1928 [von Neumann] had produced a mathematical paper on how best to operate when you logically ask yourself what the other man is going to think you mean to do. In a two-person zero-sum game (i.e., one where anything I gain equals your losses and my losses equal your gain) you can in fact work out a best strategy that will maximize your potential gains or minimize your potential losses if I operate equally logically. And you will on average beat me hollow if I operate in any way illogically at all” (Macrae, 2016).

CURIOUSITY

Economics Example

"von Neumann also proved a theorem known as the minimax theorem for zero-sum games, which would later lay the foundation for the new field of game theory as a mathematical discipline" (Veisdal, 2019).

		Japanese Route	
		North	South
Allies Reconnaissance	North	2	2
	South	1	3

The Minimax Theorem (von Neumann, 1928)
The minimax theorem provides the conditions that guarantee that the max-min inequality is also an equality, i.e. that every finite, zero-sum, two-person game has optimal mixed strategies.

Atomic Bomb Motivation

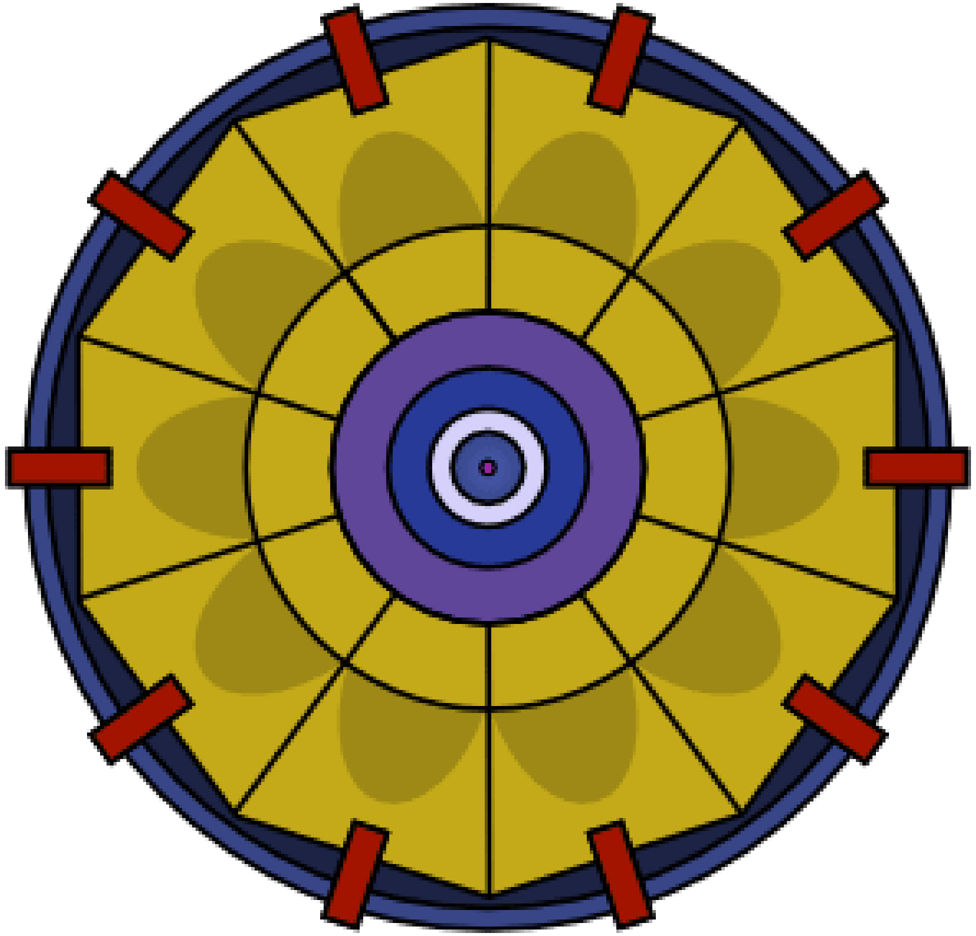
"In the winter of 1943-44 a common mood of the scientists at Los Alamos was that (a) we are doing something sinful by inventing so murderous an explosive, but (b) we have to do so because otherwise Nazi Germany might get there first; so (c) after the war we must try to internationalize these weapons (which became official American policy). **In private, Johnny disagreed with this "good guys' view" in almost every particular.**

Johnny had no doubt that weapons would be made much more murderous. Murderousness is what people in wars try to achieve. After the advent of quantum mechanics (atomic physics) in the 1920s, he saw that 'efficiency at killing people would make a quantum leap.' An A-bomb would be invented as a result of this present 1939-? war. A hydrogen bomb, which Teller had adumbrated (to Johnny, convincingly) as early as 1942, would be ready to help (with luck) to deter the next one" (Macrae, 2016).

EFFICIENCY

Atomic Bomb Example

“von Neumann‘s main contributions to the atomic bomb would not be as a lieutenant in the reserve of the ordnance department, but rather in the concept and design of the explosive lenses that were needed to compress the plutonium core of the Fat Man weapon that was later dropped on Nagasaki… von Neumann was present during the first Trinity test on July 16th, 1945 in the Nevada desert as the first atomic bomb test ever successfully detonated” (Veisdal, 2019).



Digital Computing Motivation

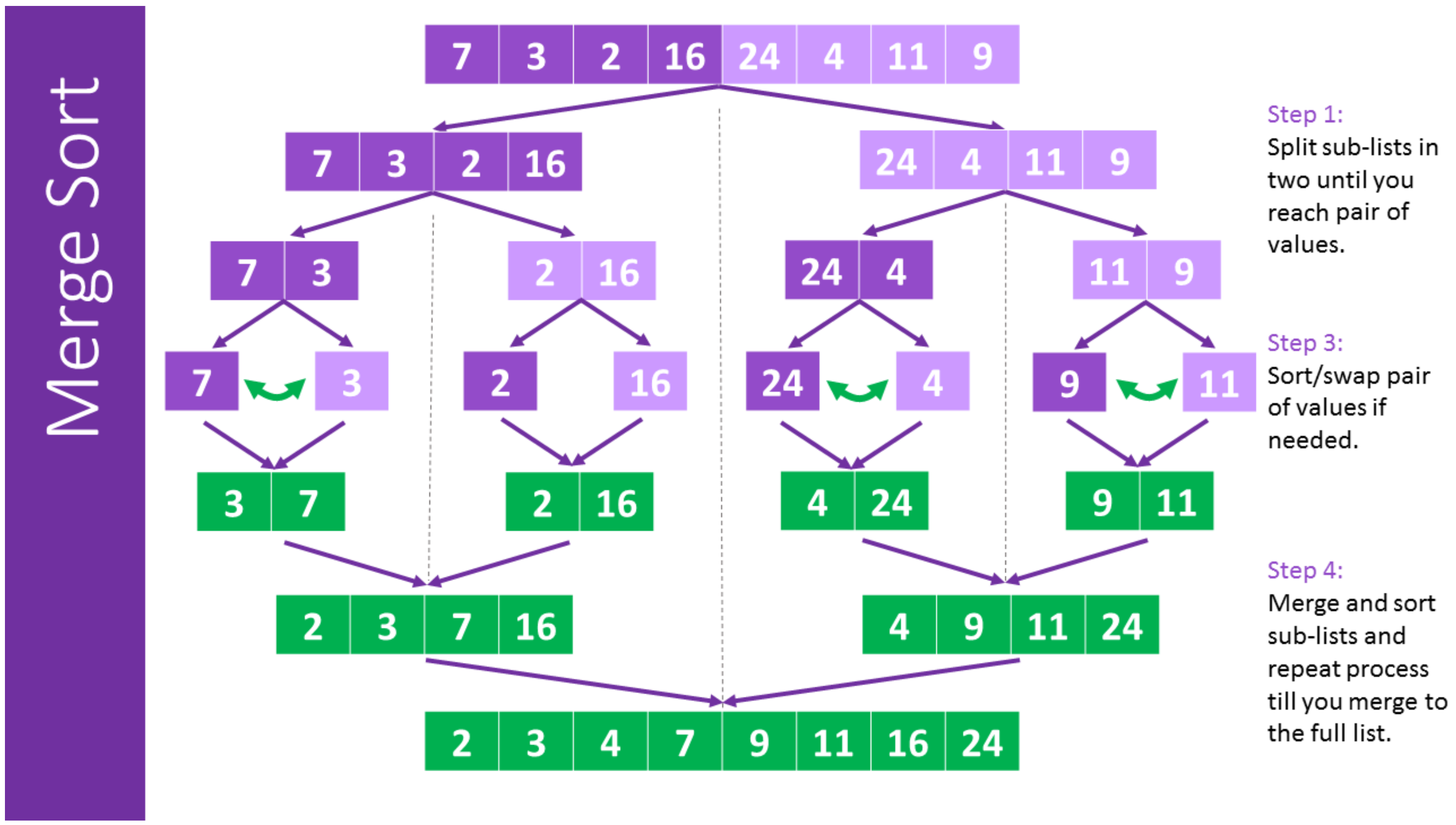
"All existing methods of approximation in mathematics, Johnny wrote to Strauss in late 1945, had been conditioned by the speeds of calculation that were possible. Now those speeds were going to become at least ten thousand times quicker. In ascending order of importance, this multiplier meant (1) that a single researcher could do calculations in a morning that would have taken him a working lifetime heretofore, (2) that research teams could do one hundred times more research projects one hundred times faster, and (3) that unimagined new fields of research would open…

Our present mathematical methods, he said, were developed for slow and purely human procedures of calculation. The electronic computer would alter the possibilities, difficulties, emphases, boundaries. It would transform the whole internal economy of computing so radically, and shift all procedural options and equilibria so completely, that the old mathematical methods would have to give way to fresh ones, based on 'entirely new criteria of what is mathematically simple or complicated, elegant or clumsy'" (Macrae, 2016).

ADVANCEMENT

Digital Computing Example

"von Neumann invented the so-called merge sort algorithm which divides arrays in half before sorting them recursively and then merging them. von Neumann himself wrote the first 23-page sorting program for the EDVAC computer in ink" (Veisdal, 2019).



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