



The Beauty and Power of Mathematics

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Mathematics is the King of Science --- Gauss

- ▶ In Science, wherever there is no mathematics, nor there is no connection with mathematics, is not reliable. --- Leonardo da Vinci
- ▶ As vast as the universe, as small as the particles, as fast as the rockets, as elegant as the elements, as diverse as the earth, as mystery as biology, our daily life, all use mathematics. --- Luogeng Hua
- ▶ Why are numbers beautiful? It's like asking why is Beethoven's Ninth symphony beautiful. If you don't see why, someone can't tell you. I know numbers are beautiful. If they aren't beautiful, nothing is. --- Paul Erdős

The book of nature is written in the language of mathematics. --- Galileo

$$e^{\pi i} + 1 = 0$$

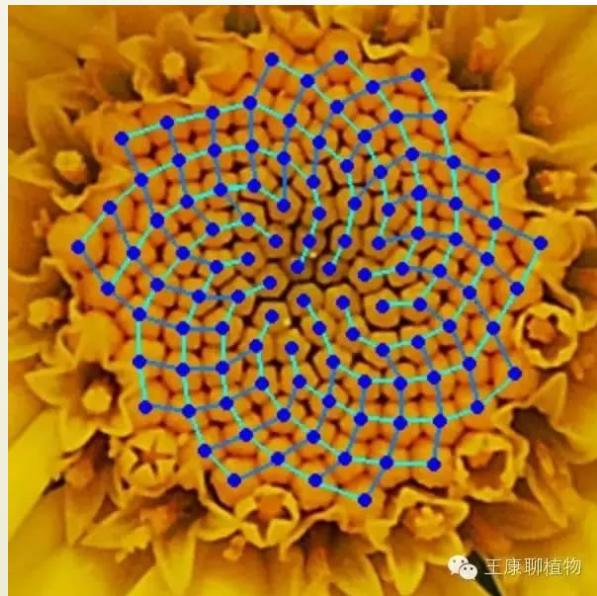
$$\int_0^1 \frac{1}{x^x} dx = \sum_{n=1}^{\infty} \frac{1}{n^n} = 1.291285997\dots$$

$$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan \frac{1}{239}$$

$$3^3 + 4^3 + 5^3 = 6^3$$

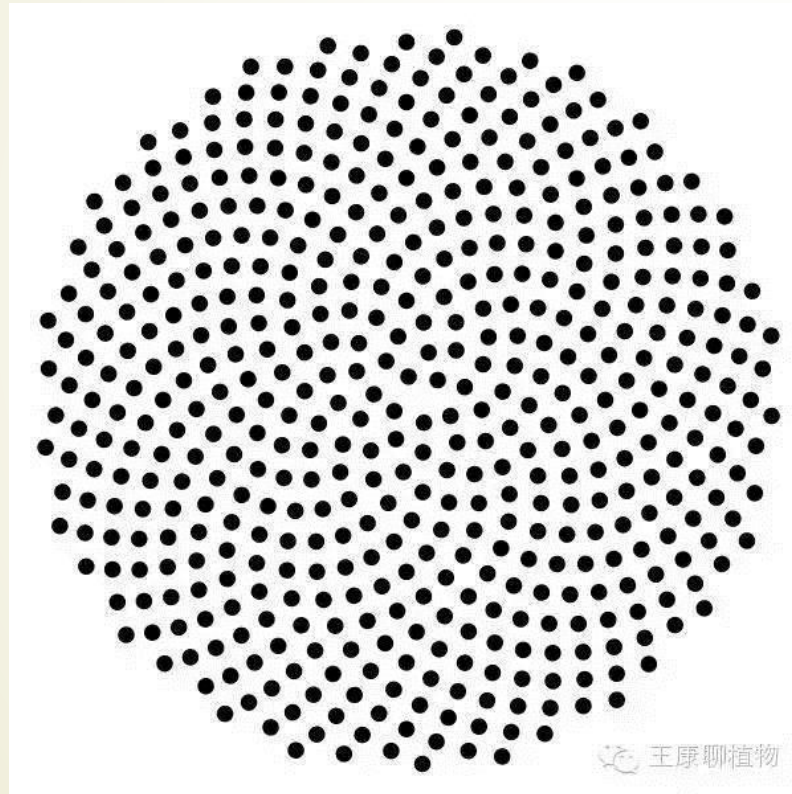
$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+\dots}}}} = 3$$

Sun Flower – Fibonacci Sequence



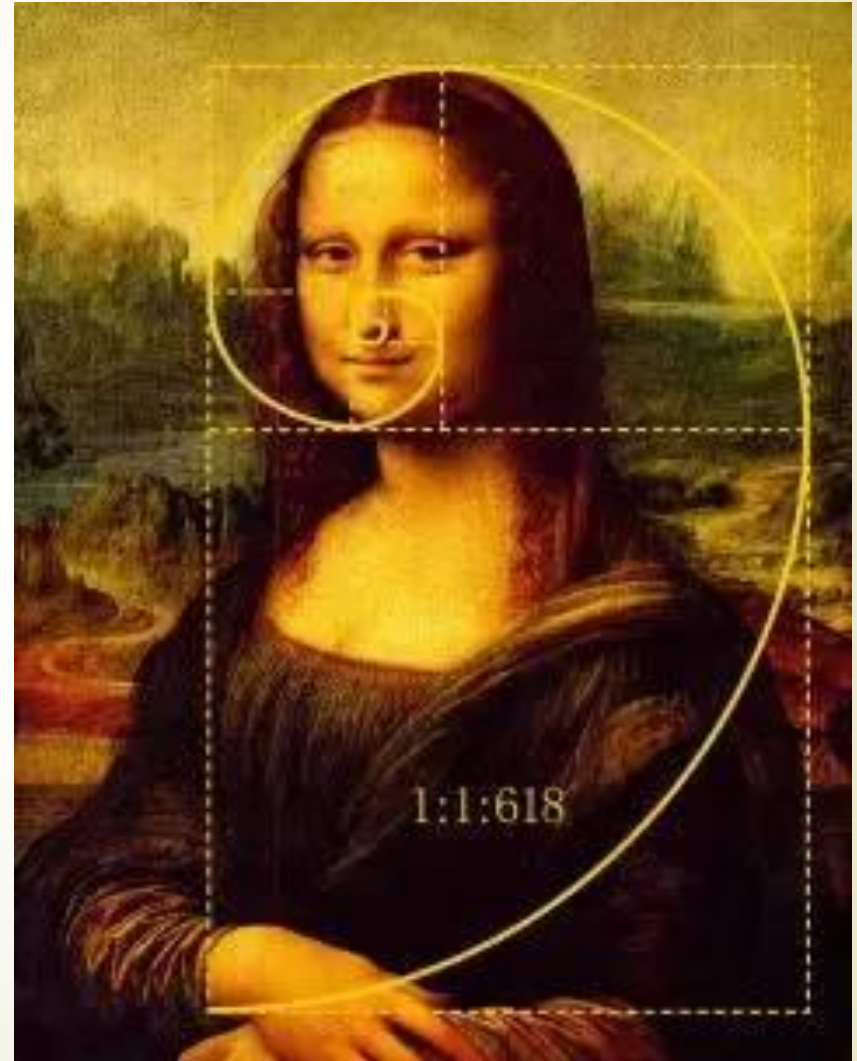
- 1,1,2,3,5,8,13,21,34,55,89,144
- Golden Ratio, $55/89=0.617977\dots$,
 $144/233=0.618025\dots$,
 $48368/75025=0.6180339886\dots$
- Green (counterclockwise) spiral: 13; blue (clockwise) spiral: 21.
- Bigger sun flower: 89, 144 or 144, 233
- Spiral angle: 137.5° , $360^\circ - 137.5^\circ = 222.5^\circ$, $137.5^\circ / 222.5^\circ \approx 0.618$

Mathematically Beautiful: Sunflower

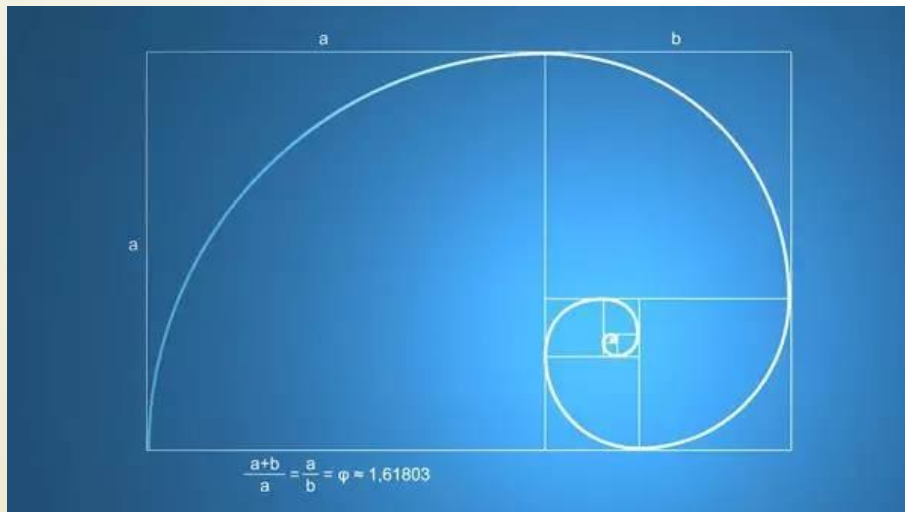
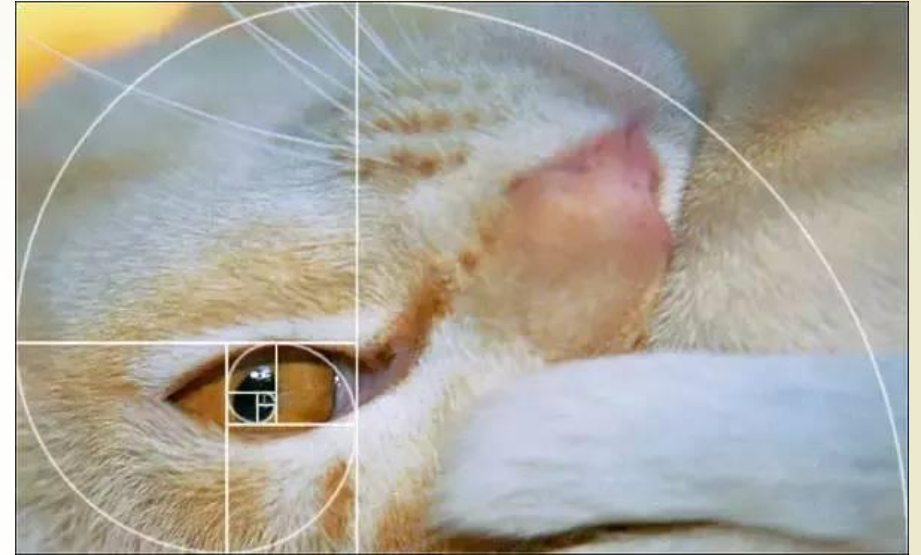
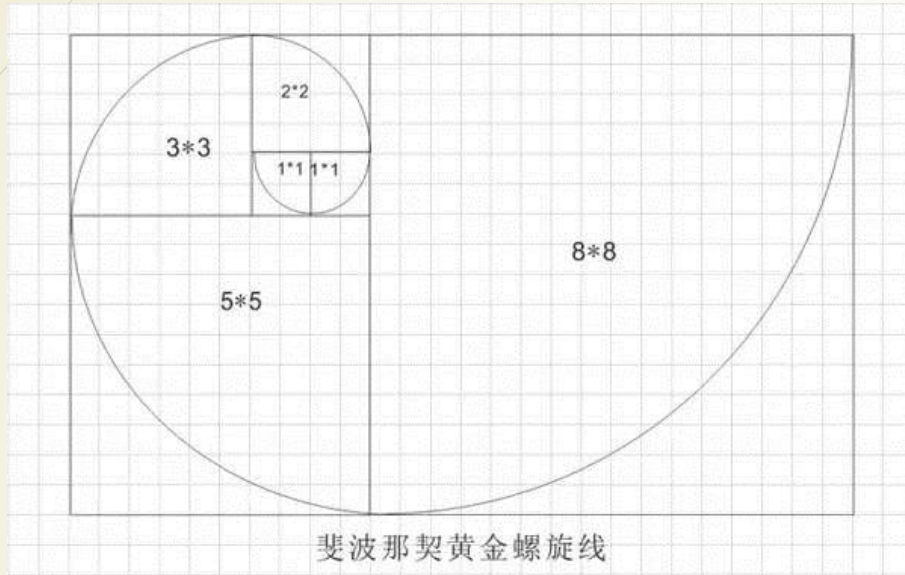


- ▶ Computer simulations show that if the angle is $>$ or $<$ 137.5° , there will be empty space between seeds.
- ▶ In the limited space of the sunflower disk, to bear enough seeds is the top priority for survival, natural selection had to make mathematically beautiful sunflower to achieve the golden ratio.

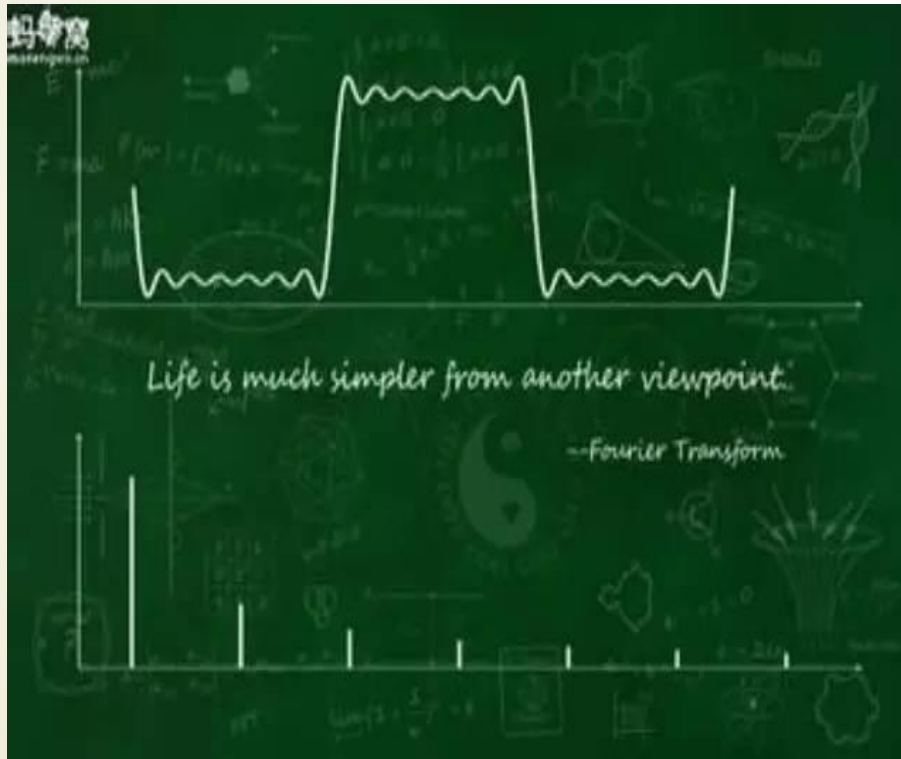
Fibonacci in Daily Life



Fibonacci sequence

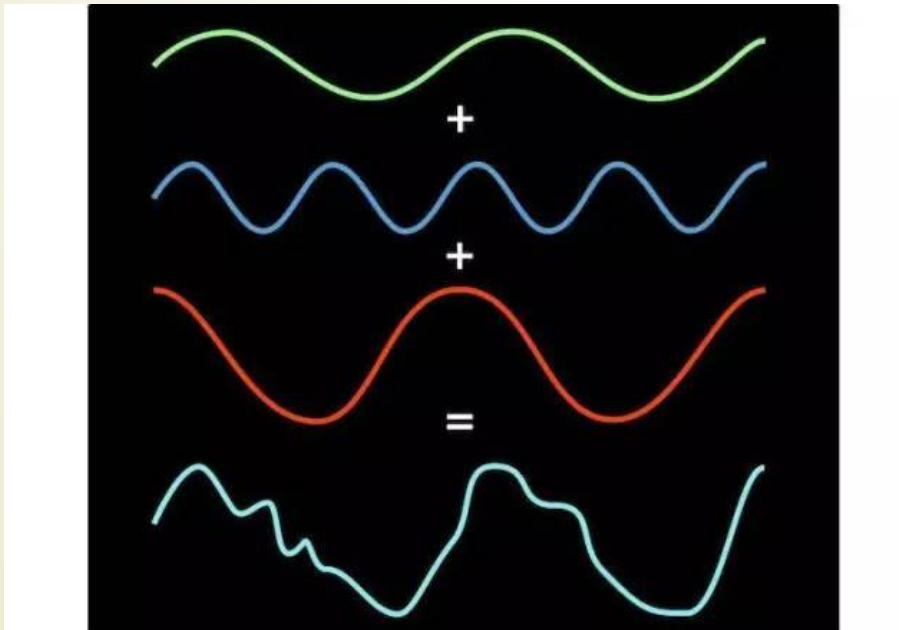
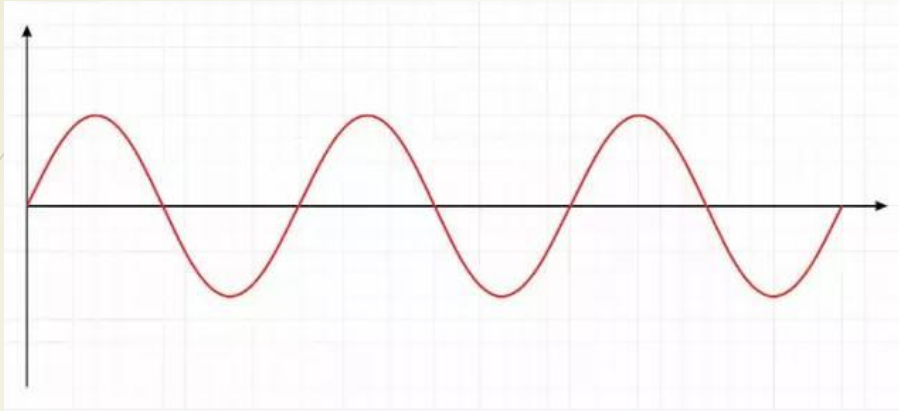


Fourier Transform: the Math behind MP3, JPEG, Siri and more



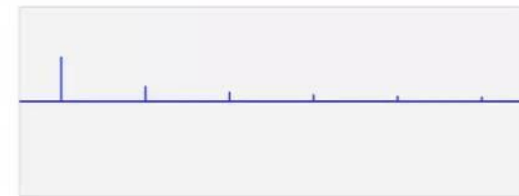
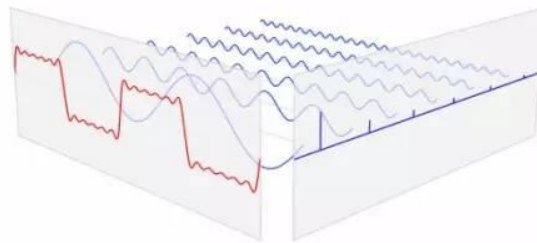
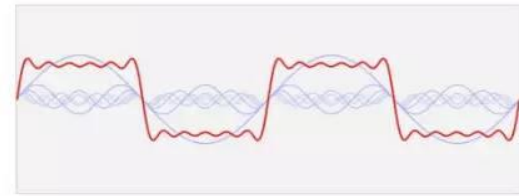
- ▶ Quantum physics, radio astronomy, MP3 and JPEG compression, X-ray crystallography, Speech recognition, PET or MRI scan
- ▶ Fourier: the first one to discover Greenhouse effect

What is Fourier Transform?

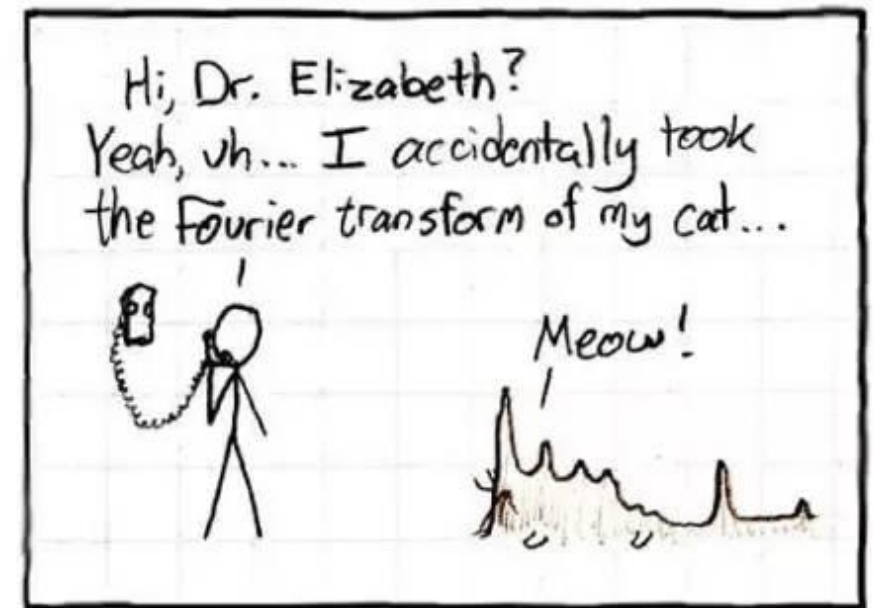


- ▶ Tuning fork
- ▶ A note from the piano: a little hammer struck a cord (for do, about 440 times/second)
- ▶ Sine curve
- ▶ How about three notes (chord)?
- ▶ Summation of three sine functions

All waveforms can be decomposed by Fourier transform to become sine and vice versa



- ▶ LucasBV in Wikipedia
- ▶ Fourier transform tells you that in a wave, the proportion of every "note" (sine curve)

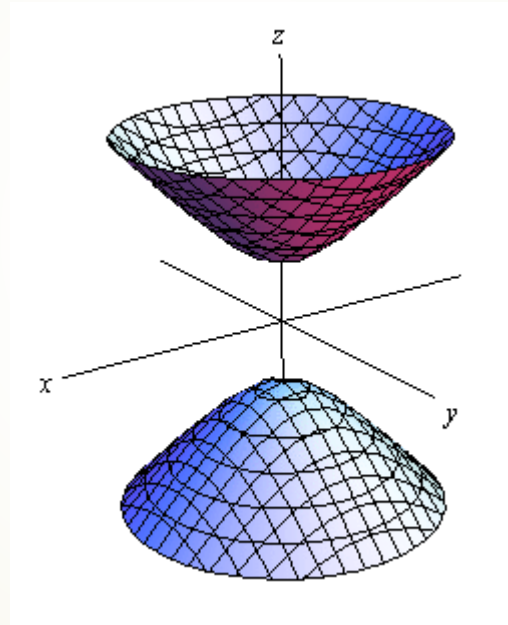
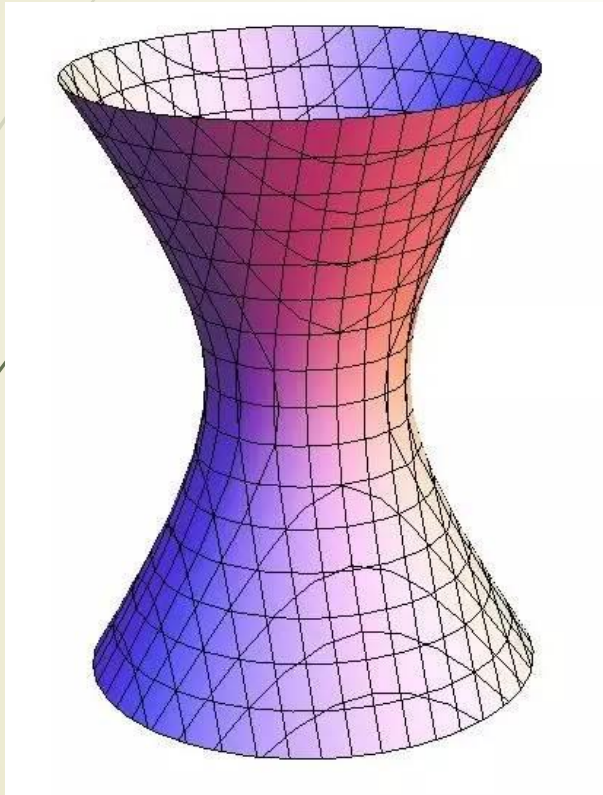




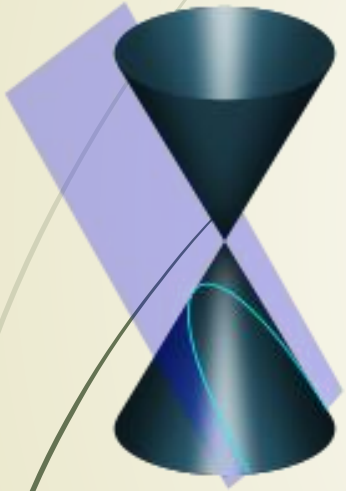
Applications of Fourier Transform --- Twitter replies from Scientists

- ▶ Submarine structures and its interaction with the flow
- ▶ Try to predict the incoming earthquake
- ▶ Identify components of distant galaxies
- ▶ Reveal the protein structure from x-ray diffraction pattern
- ▶ Signal analysis for NASA
- ▶ Acoustic research for instruments
- ▶ Improve the water cycle model
- ▶ Look for pulsars
- ▶ Molecular structure by MRI
- ▶ Decipher the painting chemicals to identify counterfeit of Jackson Pollock's painting

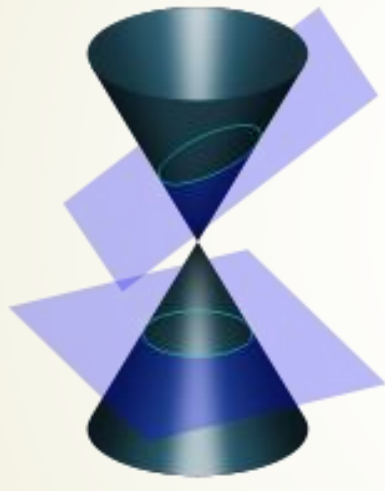
Hyperboloid of one or two sheets



Ellipse, Hyperbola, Parabola



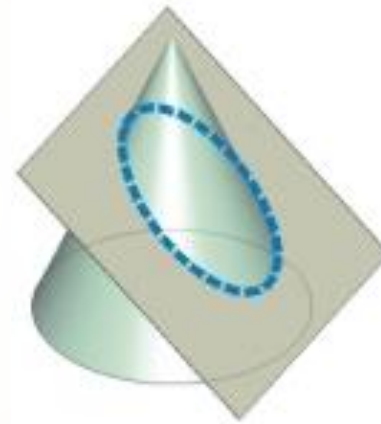
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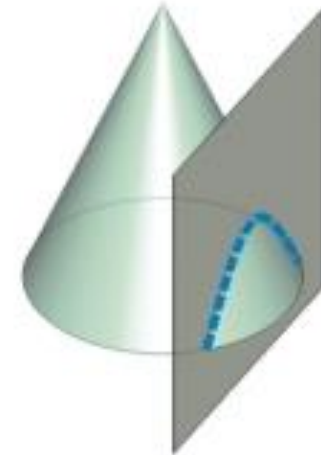
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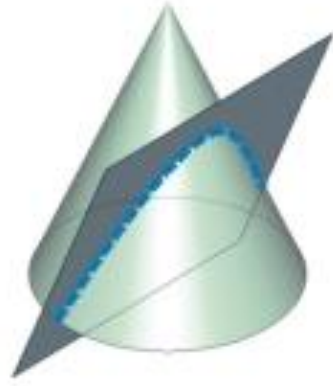
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Ellipse: angle between plane and cone axis is greater than the cone's half-angle



Hyperbola: angle between plane and cone axis is less than the cone's half-angle

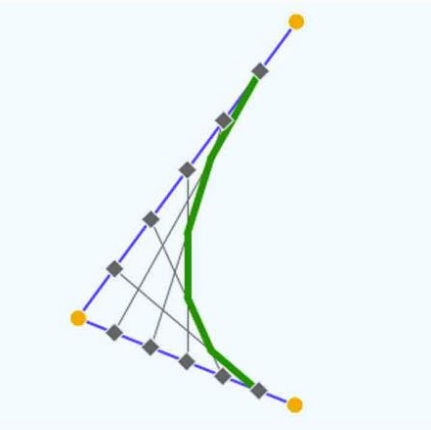


Parabola: angle between plane and cone axis is equal to the cone's half-angle

Pixar (in a box) and Math

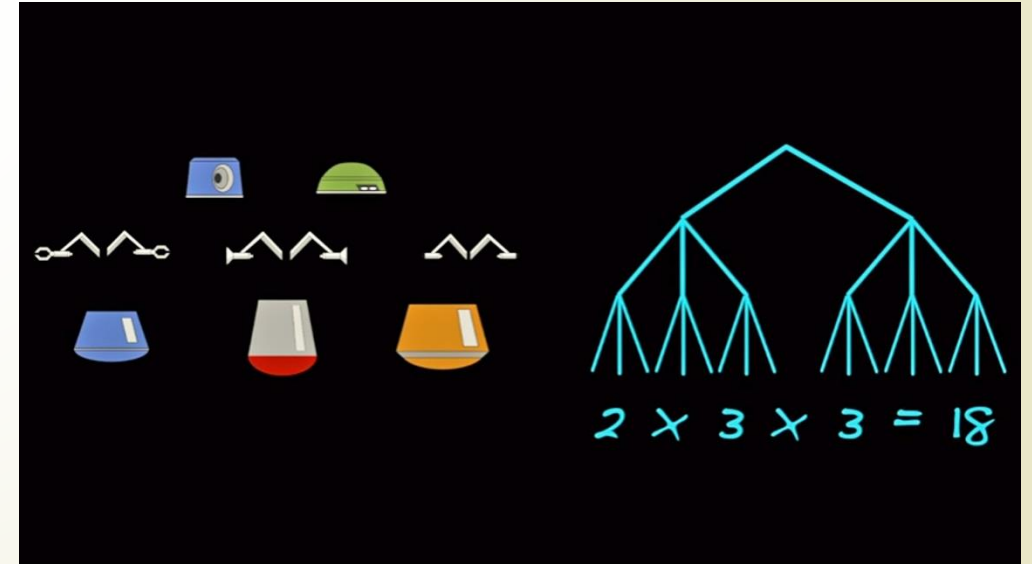
<http://ww2.kqed.org/mindshift/2015/09/03/pixar-in-a-box-teaches-math-through-real-animation-challenges/>

Welcome to Environment Modeling!



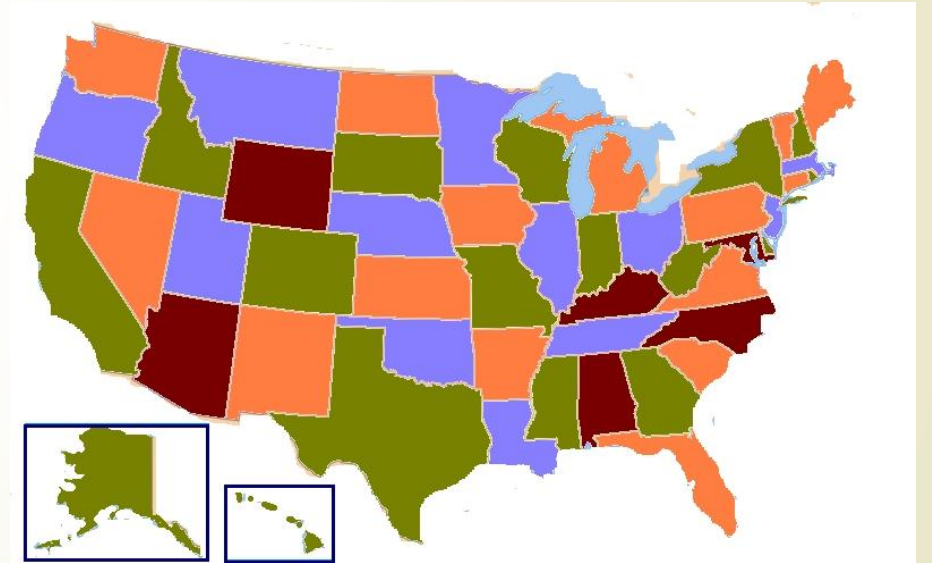
In this topic we are going to explore how realistic *blades of grass* are modeled using *parabolic arcs*.

- A combinatorics lesson shows how many kinds of robots can be made by animators with just a few parts



Four Color Theorem

- ▶ given any separation of a plane into contiguous regions, producing a figure called a *map*, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color.



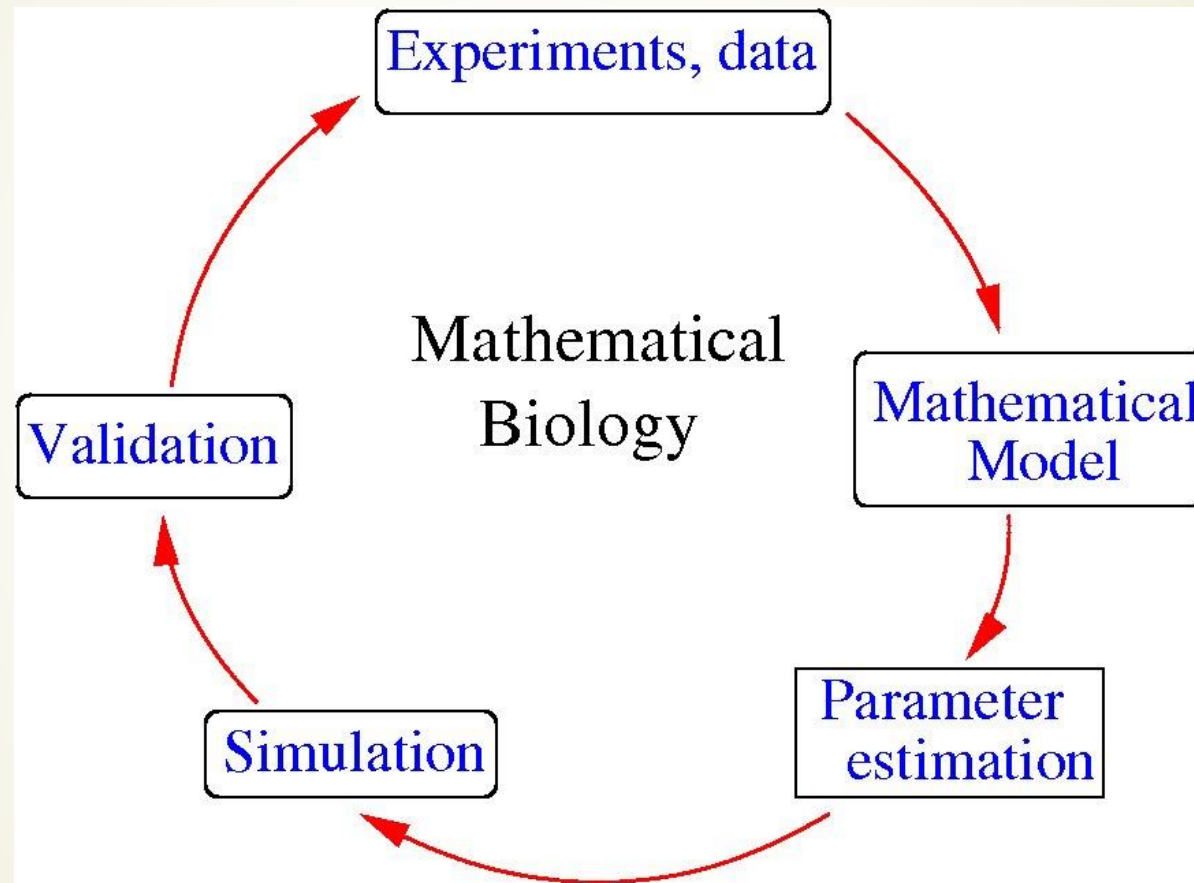
Google PageRank: solving matrix

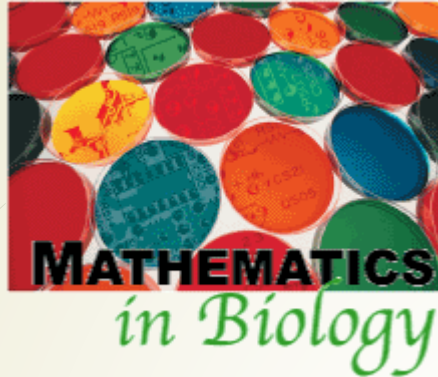
- ▶ **PageRank** is an algorithm used by Google Search to rank websites in their search engine results.
- ▶ PageRank is a way of measuring the importance of website pages.
- ▶ The PageRank values are the entries of the dominant left eigenvector of the modified adjacency matrix.

$$\mathbf{R} = \begin{bmatrix} PR(p_1) \\ PR(p_2) \\ \vdots \\ PR(p_N) \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} (1-d)/N \\ (1-d)/N \\ \vdots \\ (1-d)/N \end{bmatrix} + d \begin{bmatrix} \ell(p_1, p_1) & \ell(p_1, p_2) & \cdots & \ell(p_1, p_N) \\ \ell(p_2, p_1) & \ddots & & \vdots \\ \vdots & & \ell(p_i, p_j) & \\ \ell(p_N, p_1) & \cdots & & \ell(p_N, p_N) \end{bmatrix} \mathbf{R}$$

My research





Mathematical Biology

- ▶ In a special collection of articles published beginning 6 February 2004, Science Magazine and its online companion sites teamed up to explore **one of the hottest interdisciplinary collaborations in science** today: the **interface between mathematics and biology**. In Science, News, Viewpoint, and Review articles covered how mathematics and computation are informing biology on a wide variety of fronts, the need for incorporating quantitative approaches into bioscience curricula, and some of the uses -- and abuses -- of mathematical models for biological systems.
- ▶ <http://www.sciencemag.org/site/feature/misc/webfeat/mathbio/>



Computational Systems Biology



- ▶ **Systems biology** is the study of the interactions between the components of a biological system, and how these interactions give rise to the function and behavior of the system as a whole.
- ▶ These studies use **computational analysis** to generate hypotheses about the mechanistic behavior of the underlying biological system, to complement traditional wet-lab techniques.

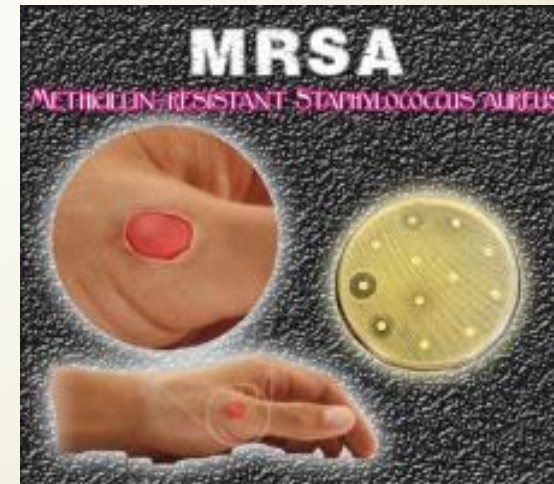
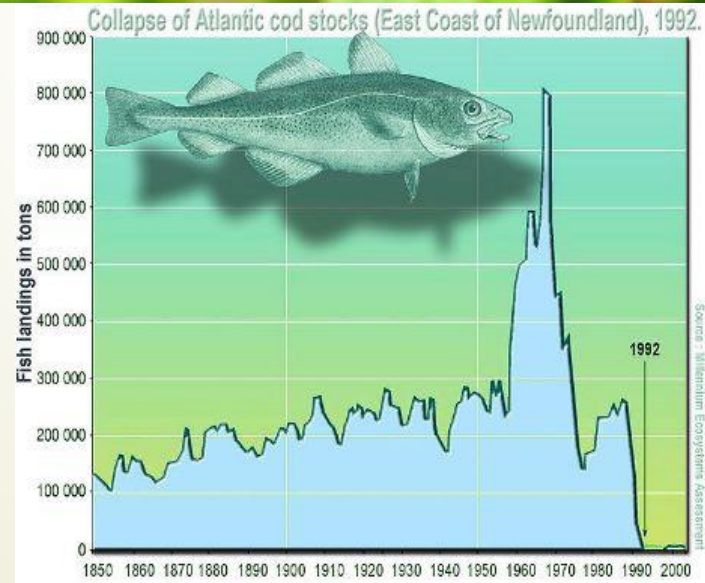
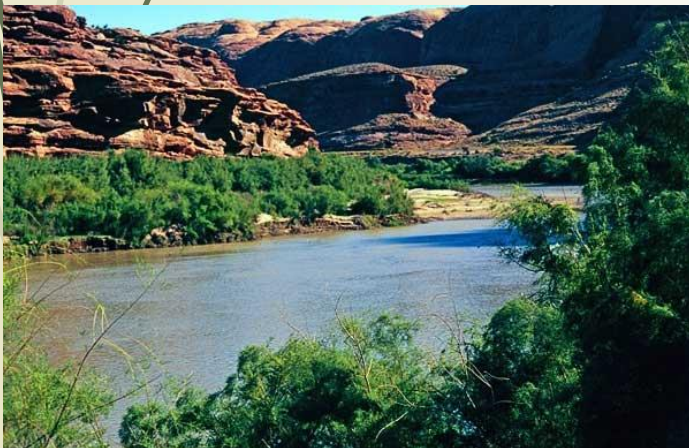


Optimal Control



- ▶ **Optimal control theory** is a branch of mathematics developed to find optimal ways to control a dynamic system. Generally, the optimal control problem consists of an objective functional, a dynamic system and the control(s). The controls enter the dynamics in a variety of ways as coefficients, boundary terms or sources.
- ▶ My research focuses on understanding the **spatial and temporal patterns** that arise in **dynamic biological systems** and when possible, finding the **best way to control** the system.

Selected Previous Projects





Mathematical Models – My Research

- ▶ Population Dynamics
 - ▶ Disease Modeling
 - ▶ Natural Resource Management
 - ▶ Cell Biology
 - ▶ Optimal Fishery Harvesting
 - ▶ Control Rabies in Raccoons
 - ▶ Invasive Species
 - ▶ Disease Modeling: MRSA, Malaria
 - ▶ Pest Control, Bio-Control
 - ▶ Non-Timber Forest Harvesting
 - ▶ Ticks and Lyme Disease
 - ▶ NF- κ B Signaling Pathway and Cryptococcus Neoformans
- 



Millennium Problems

- ▶ Yang–Mills and Mass Gap
- ▶ Riemann Hypothesis
- ▶ P vs NP Problem
- ▶ Navier–Stokes Equation

This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

- ▶ Hodge Conjecture
- ▶ [Poincaré Conjecture](#)
- ▶ Birch and Swinnerton-Dyer Conjecture