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Experience and Lessons Learned from Using SIMIODE Modeling Scenarios

Wandi Ding, Ryan Florida, Jeffery Summers, Puran Nepal and Ben Burton

Abstract: We share our experience and lessons learned from using Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations (SIMIODE) modeling scenarios in our Differential Equations I class at Middle Tennessee State University. Specific projects with Python codes are presented. Discussions are brought forth on how to "best" teach differential equations with modeling approaches while maintaining the balance with the theory. Python notebooks are attached in the Appendix and available at GitHub.

Keywords: Differential equations, modeling scenario, SIMIODE, logistic equation, SIR model

1. INTRODUCTION

Undergraduate research is classified as a "high impact" practice that affects the success of students. Studies [6] have shown that students who are engaged in undergraduate research are more successful during and after college in terms of problem solving, critical and independent thinking, creativity, intellectual curiosity, disciplinary excitement, and communication skills. Also, it has been shown that students from underrepresented groups who engage in research have improvements in grades,

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retention and graduation rates, and motivation to succeed in graduate school [6]. The latest curricular national standards from the 2015 Committee on the Undergraduate Program in Mathematics (CUPM) [8] state that every student

> … majoring in the mathematical sciences should work, independently or in a small group, on a substantial mathematical project that involves techniques and concepts beyond the typical content of a single course,

and should present the work in written and oral form.

Middle Tennessee State University (MTSU) has a strong record of supporting undergraduate research. The University's Undergraduate Research Center fosters undergraduate research by providing resources for students and faculty, including training and grants. The University is home to the TN STEM Education Center, which aims to improve STEM education at all levels, and provides support for the development, management, and evaluation of educational programs at MTSU and beyond.

However, the level of undergraduate research being done in our department is low, and one of the suggestions from an external review [7] is to recruit students for research. One way to get students to start doing research is to include student projects in certain courses. Students who are excited about their course project often are eager to follow that with an undergraduate research project.

A report [3] from the Mathematical Association of America's working group charged with making recommendations for the undergraduate curriculum in differential equations indicates a dramatic transformation over the past 20 years due to the wider availability of computer resources. However, there is no set curriculum for this course. The major change in the Ordinary Differential Equations (ODE) courses has been primarily motivated by the fast development of the computer. This new computational approach can incorporate modeling/research projects involving real-life nonlinear systems of ODEs that arise in various science and engineering applications. These projects can be team or individual assignments, and usually require an extensive written report or oral presentation. Another advantage of this computational/modeling approach is that it allows for the inclusion of topics that have not usually been treated in an introductory ODE course, e.g., phase plane analysis, visualization of solutions of systems of differential equations; and a quantitative approach to understanding the system of nonlinear systems that can not be solved analytically.

Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations (SIMIODE) [10] is an open community of teachers and learners using a modeling first differential equations teaching approach in an original way and provides extensive modeling scenarios. We participated in a SIMIODE MAA-PREP Workshop in 2015 and have been trying to incorporate and design SIMIODE modeling scenarios in our Differential Equations I course.

"How can we combine all the resources and the campus wide support of undergraduate research activities to enhance the development of our Differential Equations course?" is a central question we address here.

Differential Equations is a mid-range course in the mathematics curriculum at MTSU. It is a "transition" course from lower-level "methods" courses to upper-level "theoretical" courses. It is a nice blend of theoretical/pure mathematics and practical/applied mathematics. It has a diverse student body: mathematics, physics, computer science, aerospace, engineering technology, our new program on mechatronics engineering, and a small group from biology and chemistry. Therefore, we use this course to expose and attract students to participate in small and large-scale research projects.

We obtained internal funding to create a Mathematical Modeling Course Using Differential Equations and Statistics in 2014 with Biological Applications to further explore our modeling approach. We also submitted an NSF REU proposal about using computational modeling and simulation in applied sciences in fall 2017. We are making progress toward our goal.

We reflect on and summarize our effort, experience, and lessons learned using SIMIODE modeling scenarios from our Differential Equations class, in order to both share with the community and seek feedback to improve our modeling approach, and ultimately our undergraduate research.

This paper is organized as follows: Project descriptions and examples appear in Section 2. There we highlight SIMIODE modeling scenarios on Disease Spread Using M & M (with and without Death & Immigration) [11–13], Logistic Population Growth [14], Ebola Modeling and Control [9], Fish Harvesting [4] and Mass-Spring Oscillator/Pendulum [1, 16]. Other modeling scenarios such as Malaria Control [2] and Kinetics-Chemical Reaction [15] are briefly mentioned. We discuss the progress our students have made. Section 3 includes reflections on the effectiveness of using a SIMIODE modeling approach, and it concludes with some final thoughts and discussion.

2. PROJECTS DESCRIPTIONS AND EXAMPLES

We implemented a Python notebook for all the code our students generated for each project. It is included in the Appendix and available at GitHub https://github.com/wdingmtsu/Master.git. The reason students chose Python is that our CSCI 1170 - Computer Science I class has switched to Python, due to the high demand from industry and the scientific community. Python is also free and open source, so our code can be shared with a wider community, especially those who do not have access to Matlab, Mathematica, or Maple.

We present four combined projects with details and discussion. Additional information is provided in our Python notebook. The benefit of a modeling approach is that it is a learning cycle approach that includes experiment/data, mathematical modeling, parameter estimation, simulations, and validation. The students repeat this cycle to enhance the model and their results. It naturally incorporates the inquiry-based learning and student-centered learning approaches.

The learning process of using a modeling approach is that students are not able to get the answers in one shot. It is a recursive process, progress and improvements are made at each step. The intriguing and challenging part is how far students can go to explore the subject and connect the dots in between. They gradually get used to open-ended questions and try to come up with the "best" solution using a trial-and-error approach with team effort. It is a character-building process. Students cannot give up easily, they need to enhance and improve each time they revisit and finally present and share their results in a written or oral form. We highlight four combined projects: Disease Spread Using M & M (with and without Death & Immigration) and Logistic Population Growth [11–14], Ebola Modeling and Control [9], Fish Harvesting [4], and Mass-Spring Oscillator/Pendulum [1, 16] in more detail. Other modeling scenarios such as Malaria Control [2] and Kinetics-Chemical Reaction [15] are briefly mentioned.

2.1. Disease Spread M&M (with and without Death & Immigration) and Logistic Population Growth

After students learn about exponential population growth, it may not be so natural for them to derive the logistic population growth model. The logistic equation is more complex, and distinct features have been embedded intrinsically.

Winkel [11–13] offers a modeling opportunity in which the phenomenon of the spread of disease can be described by one differential equation, the logistic equation, which models the number of infected individuals, with and without death and immigration [11–13]. Students: (i) toss M&M candies to develop their own data of the spread of disease; (ii) record, collect, and plot the data; (iii) set-up the exponential and logistic differential equations to model the disease spread and explore which one fits the data; (iv) understand the dynamic behavior and develop strategies to estimate the parameters; and (v) try to make predictions using the model. It is the full cycle of scientific research, of which students have little experience in a traditional mathematics class.

In general, students do well in steps (i) to (iv). To our surprise, the students estimate the parameters in four different ways. They go beyond the Least Squares Method. There is a diverse analytical and computational background among the students in the Differential Equations I class at MTSU.

Some students use the derivation of the solution of the logistic equation to figure out a “linear” relationship to get the parameters, see photos and details in [5]. Those who are good at programming and numerical methods can estimate the parameters using: (i) the simulated annealing method; (ii) Newton’s method with bisection; or (iii) the gradient descent method. These are usually the physics majors. When we prepared for the Python notebook, one student lost his original data, he managed to write a Python code to generate the data without doing the experiment. That opens a door for more computer-fluent students to combine the power of programming and mathematical modeling for their projects and share the results with their peers. We can see the enthusiasm in the students for applying what they learned in other courses (e.g., scientific modeling methods) come-up with a different method once an old one is presented. It stretches their comfort zone to research beyond what the project requires, thus giving momentum for the students to do the following projects and extend some to research projects.

The files BiNew_RF.ipynb, growth_rate_R_Parameter_Estimation.ipynb, GradientDescent.ipynb and Sim.ipynb are available at https://github.com/wdingmtsu/Master.git and also in the online Appendix. Since the students took a great amount of time investigating this project, they can easily adapt the codes for Logistic Population Growth [14].

In class, students wondered if the population is too small to sustain the population growth in the beginning, which leads to a discussion about the logistic growth equation with Allee Effect that is not usually covered in this context. I believe the modeling approach encourages students to think critically about the model they constructed, to understand the assumptions they made, and to initiate a lot of “what-if” situations and come up with satisfying answers.

This is one of the favorite modeling scenarios for students, it is an eye-opening project especially for beginners who have not previously seen a modeling project. They enjoyed the whole process of carrying out an experiment, collecting data, curve fitting, and parameter estimation. It also reinforced the procedure of the analytical solution of solving logistic differential equations. It is a fun project with a lot to learn and share. This is the first time students see the beauty and power of differential equations using a modeling approach. We have not done all of these activities in one “setting.” Students take time to come up with limiting factors and understand the logistic differential equations, as compared

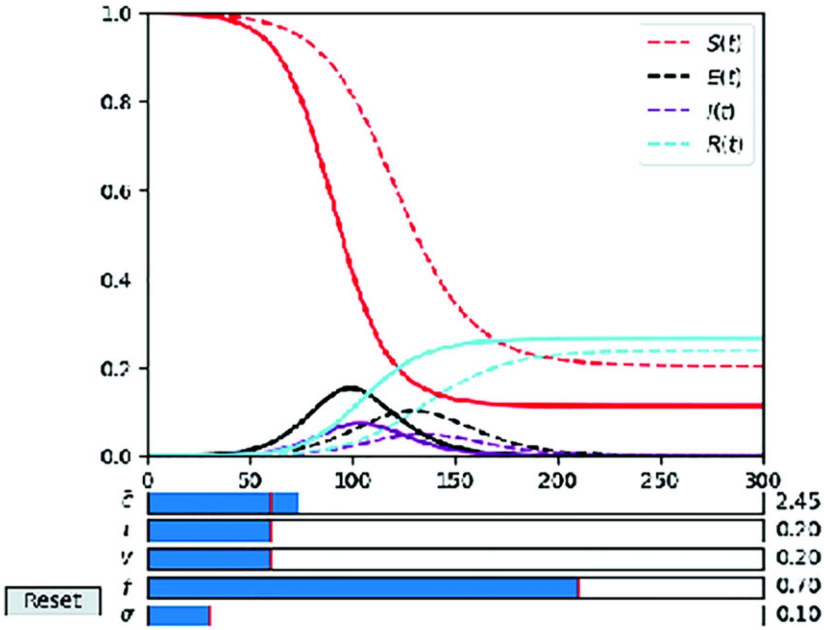

Figure 1. Ebola modeling with slider: Vary parameters $\bar{c}, \tau, v, f, \sigma$.

with exponential growth. This is an early project to build in limited growth and the students struggle with this problem. A lot of trials and conversations gradually lead to some form of the logistic differential equation and students feel confident about the group effort. It depends on the analytical and numerical skills of each student to come up with different approaches for parameter estimation of the growth rate r. Students reflect on these different approaches to get better insight into the modeling process. Each individual is an independent thinker, and the group work made a good final product. Students became encouraged to take a further numerical analysis course after seeing three numerical methods. I am very satisfied with the depth our students have experienced in this project.

2.2. Ebola Modeling and Control

There is always the challenge to learn and understand systems of ordinary differential equations. After introducing the standard SIR model for modeling the spread of a disease, students struggle to get a full picture about the system, e.g., how the dynamics of each population changes over time, what are the effects of different parameter values affecting the system? Do the initial values play a role for the outcomes? Is the system stable? Those answers are not easily handled from a pure analytical point of view.

Payner et al. [9] have an Ebola model that consists of susceptible, exposed, infected, and recovered populations. The set of parameters reasonably describes the dynamics of the Ebola virus outbreak in Liberia, as of November 10, 2014. The Liberian government is seeking international funding to aid in combating the Ebola epidemic and it can allocate the money towards three different programs: Protection and Awareness Program, Contact Tracing Program, and Treatment Units. Each program has a different effect on the coefficients in the model. The projects asks for Part I - Equilibrium and Stability Analysis, Part II - Numerical Analysis, and Part III - Sensitivity Analysis. This is a very challenging project since not all the topics are covered in our Differential Equations I class.

Our students could not finish Part I and III completely, but they did an excellent job modeling the dynamics of the susceptible, exposed, infected, and recovered classes, exploring the sensitivity of the parameters and the possible control strategies. First, students need to learn how to program in a system setting, as they have been used to only one equation. Second, there are five parameters in the system that could vary and change the behavior of the outcomes. What are the effect of changing one or more parameter(s) to affect the population dynamics? It is very time-consuming to modify one parameter at a time to compare and interpret the different dynamics. After exploring the dynamics of the system of four differential equations, the students create a slider for the system. You can slide/change those five parameter values and see the corresponding outcomes immediately and compare with the results of the default parameter values. It permits this for all five parameters and goes beyond what is required by the project description. It is a great tool for teachers to use to illustrate how different parameter changes affect the outcomes of the various populations. See Figure 1 for a snapshot of changing one parameter compared with the default values. The students are very interested in sliding the bars and examine the different curves corresponding to the changes. It is not the required sensitivity analysis, but it goes beyond what I expected. The code: Ebola.ipynb is at https://github.com/wdingmtsu/Master.git and in the Appendix. Later on, students can use this code to do similar analysis for other systems of differential equations. They believe the visual representations yields insights that often remain hidden from strictly analytic approaches. There are similar features in Mathematica, but Python is free and open source, our students have the passion to create and share the useful codes and also improve my teaching.

2.3. Fish Harvesting

We designed a new SIMIODE modeling scenario of harvesting at a fishery using logistic growth with harvesting of Atlantic cod proportional to the stock available. That was a product directly from the SIMIODE MAA-PREP Workshop in which I participated in the Summer of 2015. We used the data from Georges Bank and extended the model to a partial differential equation [4].

It is a semi-linear, partial differential equation and can be solved by the separation of variables technique. Students need to employ the partial fraction decomposition technique when solving it analytically. Students have experience solving one or more differential equations at this stage.

In this project, one of the challenges is how to deal with the real data. The data given does not reflect constant harvesting rates as used in the model. Students proposed to have harvesting as a function of time, which leads to a discussion about using optimal control theory to tackle the problem. At this stage, students are not equipped with this theory; however a few students showed great interest in this problem and they proposed an optimal fishery harvesting problem with the objective to maximize the yield while minimizing the cost of harvesting. They will continue to study this problem as a research project. Another challenge is how to solve the partial differential equation when space is considered. This went beyond the regular Differential Equations I class. Again, a few students are interested in continuing their effort on this fishery application and we see a research project on the horizon.

Although students were not able to fully finish this project, I am delighted to see research projects coming out of it. This is one of our primary goals, to use these in-class modeling scenarios to lead to undergraduate research projects. A few students also linked this optimal control problem to the Ebola [9] project that they could not finish completely and set the goal to learn optimal control theory and numerically solve the equations. I would like to think that this self-driven study attitude is a result of adopting the modeling approach in our Differential Equations classes.

2.4. Mass-Spring Oscillator/Pendulum

In our Differential Equations I course, the chapters on second order, linear, constant or variable coefficients, homogeneous and non-homogeneous differential equations are a big challenge for most students. We teach the undetermined coefficients method, variation of parameters, and Laplace transforms techniques to solve them. It is one of the most difficult parts of the curriculum. Students without a physics background have a hard time connecting and understanding the formulas with the different dynamical behaviors of the solutions. Winkel [16] designed a pendulum modeling project to understand the simple pendulum with and without resistance. It is a great tool for students to explore periodic and damping behavior of the pendulum and numerically examine the phase plane. Upon finishing this projects, students go on to explore [1], the military applications of spring-mass systems to further understanding the second order, linear, constant coefficients, homogeneous and non-homogeneous differential equations. The files PendulumWithOutResistence.ipynb, ResistancePendulum1.ipynb and ResistancePendulum2.ipynb are at https://github.com/wdingmtsu/Master.git and in the Appendix.

Winkel [16] gives a good preparation for students to understand the full mass-spring oscillator. Students can see the harmonic motion and the oscillatory behavior and get used to reading the phase plane plots. Then they dive into the full equation. Without this modeling scenario, students have a hard time visualizing how the changes in the friction and/or stiffness affect the dynamics of the mass-spring oscillator. After observing the curves, students go back to the equations and the analytical derivation to better understand the oscillator and in general, the second order, linear, constant coefficients, homogeneous and non-homogeneous differential equations. Students appreciate the hybrid approach as a new learning tool. They even proposed the question of “how about the variable coefficients?” We did not have time to further explore this complex situation, but again that is a start of a research project.

2.5. Other Modeling Scenarios

Culver [2] has a project on Malaria control. Suppose you are the Battalion Medical Service Officer and will deploy to Liberia within the next 45 days to assist the Liberian military in its construction of medical treatment facilities. Your commander has asked you to analyze malaria preventive measures for the soldiers in your unit. The project requires understanding pharmacokinetics. Your task includes analyzing the anti-malarial drug dosing regimen for the battalion. The primary concerns are how soon to start treatment before the battalion arrives in Liberia and the potential risks if soldiers miss one or two of their scheduled doses. The students are able to numerically solve the equations and plot atovaquone and proguanil concentrations in the blood after any days with 0, 1, or 2 missed dose(s). The code Malaria-Control.ipynb is at https://github.com/wdingmtsu/Master.git and in the Appendix. Students also finished Kinetics - rate of Chemical Reaction [15]. It is included at the end of the notebook: simiode1.ipynb and in the Appendix.

The following is a sample annotation in the code from our Python Notebook. Students comment in great detail about how to use the code.

Listing 1: Sample Python Code Annotation

```
#Purpose: This is a program template that will fit a curve to any
#   data set using the method of gradient descent. Note that
#   this program does require that you make a.csv file
#   containing your data. The format of the file is specified
#   in the ReadFile function (#1) below. Also be aware that
#   this program will optimize the curve for all three
#   parameters: the carrying capacity, the steepness of the
#   curve, and the initial population. If there is a parameter
#   that you do not wish to optimize, then the easiest thing
#   to do would be to hard code your value for that right
#   before the final plot is generated. Be aware that some
#   data sets may require multiple training sessions.

* * * * * * * * * * * * READ THIS * * * * * * * * * * * * * * * *
'''
IMPORTANT: If you train your data multiple times, make sure to back up your
input file if it has valuable parameters that you like because training and accepting
the new data will overwrite your old data file.
'''
```

The full Python notebook from which this listing is extracted at https://github.com/wdingmtsu/Master.git. We welcome any suggestions and comments on the notebook. It is also offered as a PDF file and is in the Appendix.

3. CONCLUSION AND DISCUSSION

After completing the projects, students have realized that creating the model and interpreting the solution are just as important, maybe more important than simply finding a solution. Making valid assumptions to simplify the model is stressed in the classroom. Asking, “So what?” or “Is my answer reasonable?” leads students into the interpretation stage. Writing is incorporated into the process by requiring students to submit a report for each project. If time permits, students give presentations to share their results with the whole class.

When I assign the modeling projects, the students work in two-, three-, or four-person teams. I usually allow 2 or 3 weeks for students to work on the problems, and I require a formal write-up with all aspects of the modeling process. We use the format of a scientific report. If the students are willing, I give extended time for them to enhance it and make it better. About a week into the projects, I hold informal 15-minute in-progress reviews with the student teams. This requires them to have done some work (instead of waiting until the last minute), and the teams brief me on their progress. We discuss any questions they might have. It keeps the groups focused and I find that the groups try to accomplish as much as they can before the meeting. The discussions we have are very valuable, to both the groups and to me. I find that they try to explain their thinking to me, before I have graded the write-ups. I can also adjust anything that causes confusion to the students.

Student feedback has been very positive. My experience has shown that students are more apt to really learn the mathematics when they can apply it to solve real-world problems. The students feel that classes are more interesting. They also believe that their confidence levels rise after tackling a real problem. They are beginning to understand that their reasoning and computational skills are as important as their analytic skills.

We started a Teaching-Trio program at our department to discuss and enhance our teaching and it spread to the College of Basic and Applied Sciences and eventually we have a Faculty Learning Community (FLC) at the university level devoted to this program. I participated in this FLC program. During the discussion, a question was raised about maintaining a balance between the theoretical and applied nature of the materials. The issue is that students become knowledgeable about techniques of obtaining a solution and have at least a rudimentary knowledge of the underlying theory, while being able to solve real-world problems from sciences and engineering. Previously we focused on teaching students how to obtain solutions for various classes of differential equations and analyze a differential equation in order to make a qualitative statement about the solution. Now, a broader goal is to encourage students to use theoretical results, numerical methods, and a modeling approach to solve real-life problems.

We would like to point out that differential equations has traditionally been taught as a "method" course. The students are "overwhelmed" with all the details that involve lengthy algebraic manipulations, differential calculus, and the new techniques (e.g., variation of parameters). All these are challenges for students and discourage them from understanding the concept, theory, and applications of differential equations.

Our new paradigm emphasizes the modeling process (using SIMIODE modeling scenarios), as well as the analysis of the differential equations and understanding the qualitative and quantitative behavior of the solution, which incorporate the use of computer technology (e.g., Python), and writing about mathematics, not just doing mathematics. We also encourage students to present their projects and findings at MTSU Scholars Week and Undergraduate Research Conferences.

The use of computers is to help developing critical thinking skills and not be a once-and-for-all tool for differential equations. Modeling activities need to be carefully designed and incorporate the various methods of teaching to accomplish our goal. The computers can do the calculations, but the students need to do the thinking. Models bring life to students and me. We like them more than we thought we would.

APPENDIX

A separate PDF for the Python Notebook is offered electronically with this article at the *PRIMUS* journal site. It is also available at GitHub: simiode1.ipynb - https://github.com/wdingmtsu/Master.git.

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