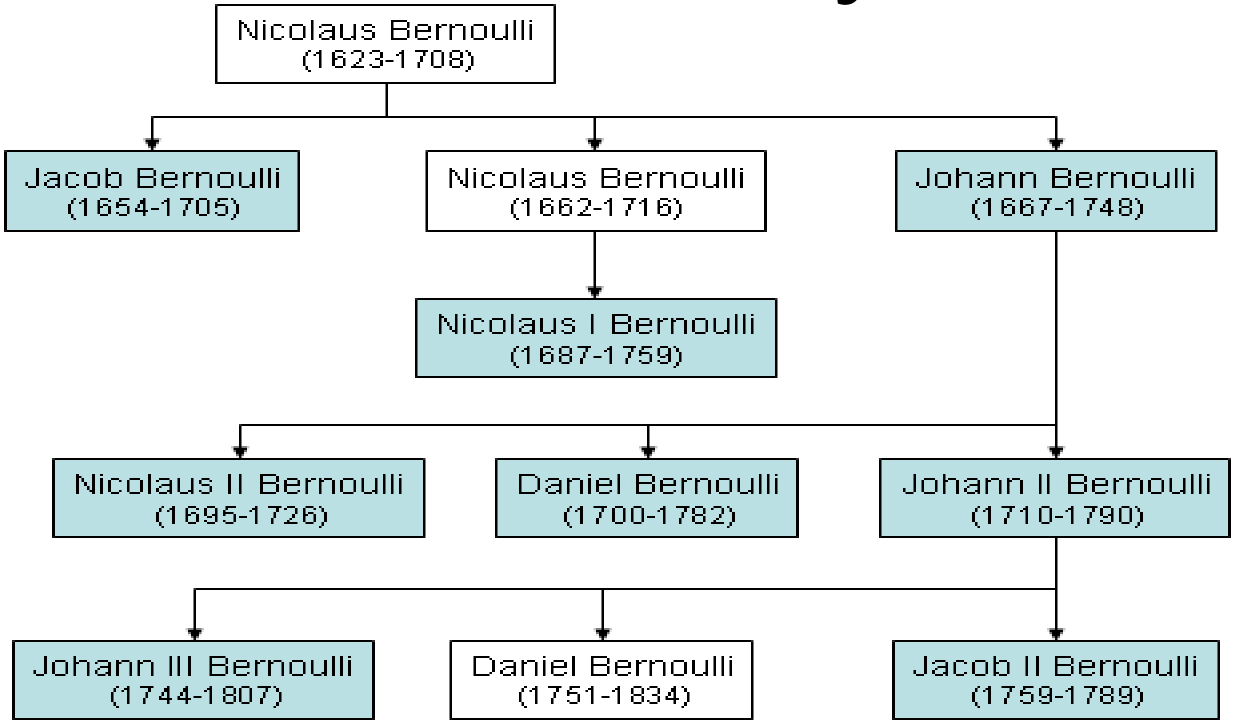
The Bernoulli Family



By Carina Vazquez

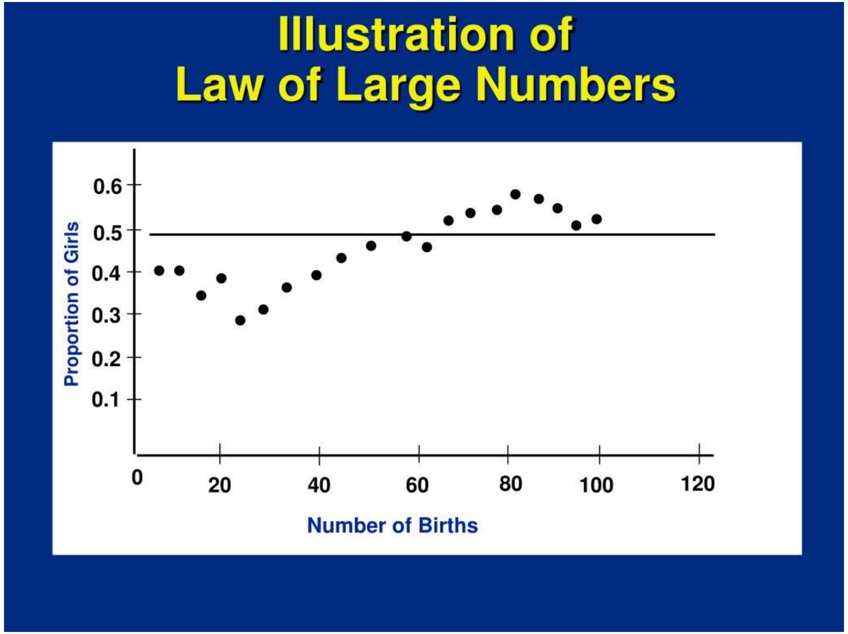
Jacob Bernoulli

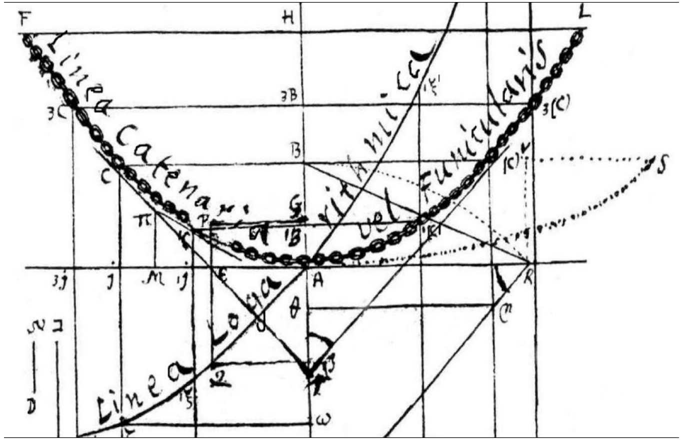
- Intrigued with mathematics, astronomy as a child.
- First Bernoulli to be recognized as a strong influential mathematician.

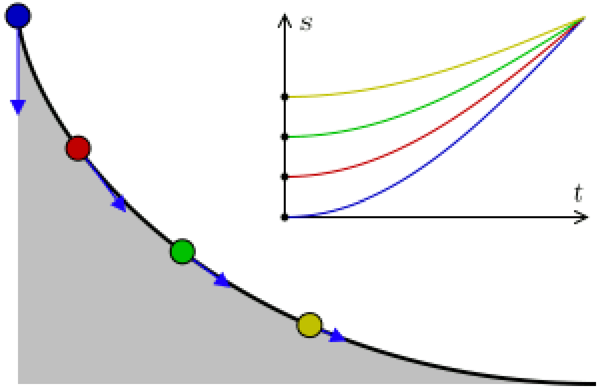
Studied calculus with Leibniz’s notes.

Contribution to Calculus

- Isochrone Curve
- Catenary Study
- Law of Large Numbers





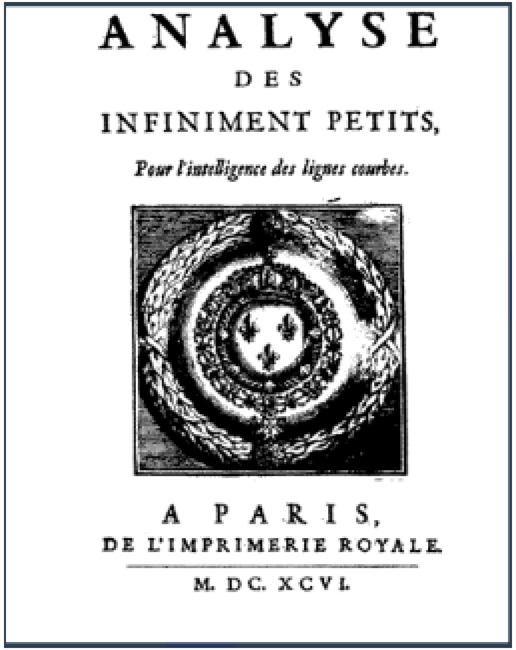


Johann Bernoulli

- Studied medicine at the University of Bale while simultaneously learning mathematics from Jacob.
- Taught calculus to Marquis de L’Hopital.

Contributions to Calculus

- Theory of Differential Equations
- Collaborated in the first Calculus Textbook


ANALYSE
DES
INFINIMENT PETITS,
Pour l'intelligence des lignes courbes.

A PARIS,
DE L'IMPRIMERIE ROYALE.
M. DC. XCVI.

Jacob Bernoulli proposed the Bernoulli differential equation in 1695.[3] This is an ordinary differential equation of the form

$$y' + P(x)y = Q(x)y^n$$

Bernoulli differential equation

-Jacob Bernoulli

$$y' + P(x)y = Q(x)y^n$$

Consider the Bernoulli equation

$$y' - \frac{2y}{x} = -x^2y^2$$

(in this case, more specifically Riccati's equation).

$$y'y^{-2} - \frac{2}{x}y^{-1} = -x^2$$

Changing variables gives the equations

$$u = \frac{1}{y}\ ,\ u' = \frac{-y'}{y^2}$$
$$-u' - \frac{2}{x}u = -x^2$$
$$u' + \frac{2}{x}u = x^2$$

which can be solved using the integrating factor

$$M(x) = e^{2\int \frac{1}{x}\,dx} = e^{2\ln x} = x^2.$$

Multiplying by $M(x)$,

$$u'x^2 + 2xu = x^4.$$

The left side can be represented as the derivative

$$\int (ux^2)'\,dx = \int x^4\,dx$$
$$ux^2 = \frac{1}{5}x^5 + C$$
$$\frac{1}{y}x^2 = \frac{1}{5}x^5 + C$$

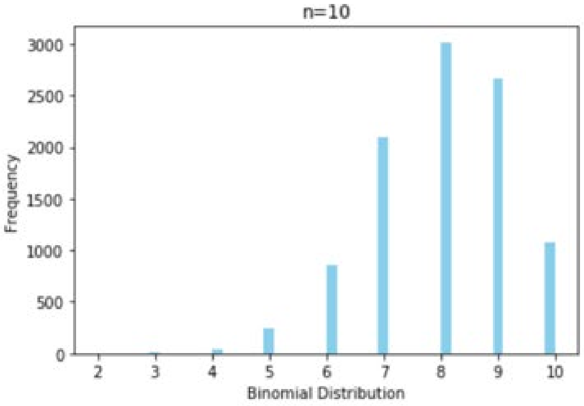
The solution for y is

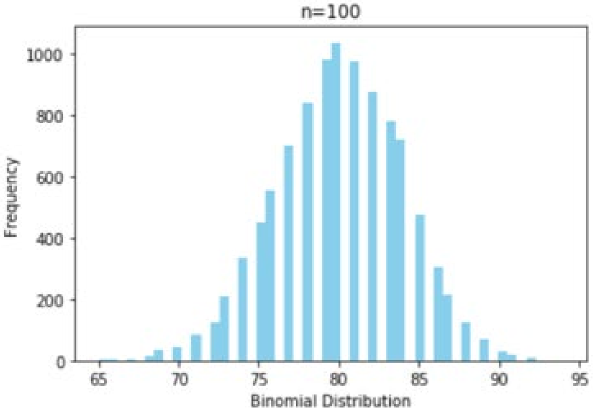
$$y = \frac{x^2}{\frac{1}{5}x^5 + C}.$$

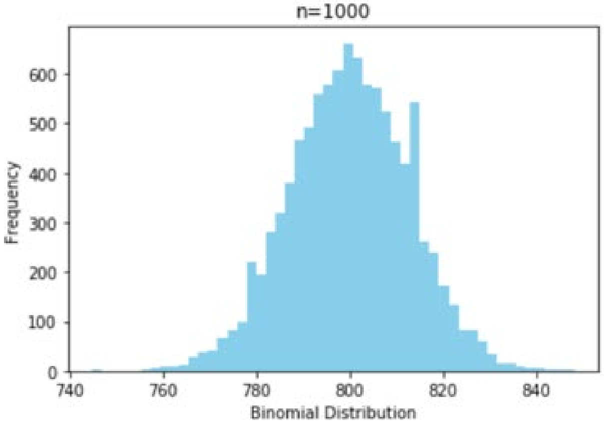
Bernoulli distribution

- Jacob Bernoulli

In probability theory and statistics, the Bernoulli distribution is the discrete probability distribution of a random variable which takes the value 1 with probability p and the value 0 with probability q=1−p. Less formally, it can be thought of as a model for the set of possible outcomes of any single experiment that asks a yes–no question. Such questions lead to outcomes that are boolean-valued: a single bit whose value is success/yes/true/one with probability p and failure/no/false/zero with probability q. It can be used to represent a coin toss where 1 and 0 would represent "heads" and "tails", respectively, and p would be the probability of the coin landing on heads or tails, respectively. In particular, unfair coins would have P =1/2. The Bernoulli distribution is a special case of the binomial distribution where a single trial is conducted. It is also a special case of the two-point distribution, for which the possible outcomes need not be 0 and 1.







Bernoulli numbers

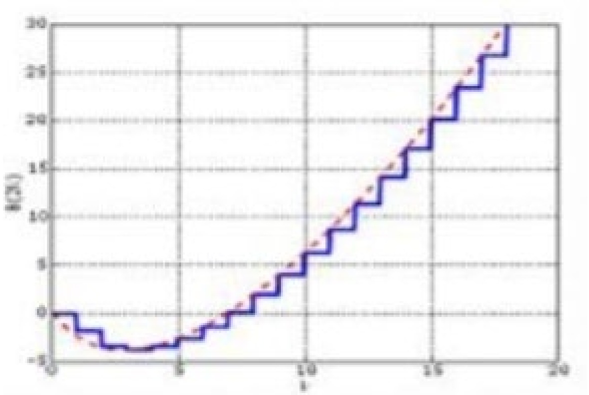
-Jacob Bernoulli

His theory of permutations and combinations sparked the reasoning behind Bernoulli numbers. Bernoulli numbers is the derivation of the exponential series. His treatment of mathematical and moral predictability and the subject of probability is what is now known as the Bernoulli Law of Large numbers.

Bernoulli numbers: $$B_m(n)=\sum_{k=0}^{m}\sum_{v=0}^{k}(-1)^v\binom{k}{v}\frac{(n+v)^m}{k+1},$$

The first few are:

B_0	=	1
B_1	=	$-\frac{1}{2}$
B_2	=	$\frac{1}{6}$
B_4	=	$-\frac{1}{30}$
B_6	=	$\frac{1}{42}$
B_8	=	$-\frac{1}{30}$
B_{10}	=	$\frac{5}{66}$
B_{12}	=	$-\frac{691}{2730}$
B_{14}	=	$\frac{7}{6}$
B_{16}	=	$-\frac{3617}{510}$
B_{18}	=	$\frac{43867}{798}$
B_{20}	=	$-\frac{174611}{330}$
B_{22}	=	$\frac{854513}{138}$



Bernoulli's principle

- Daniel Bernoulli.

The total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion, remains constant.

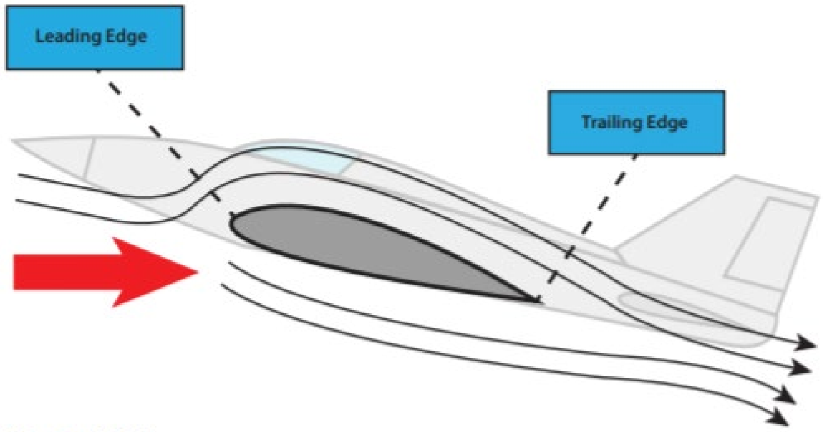


Fig. 3 Airfoil

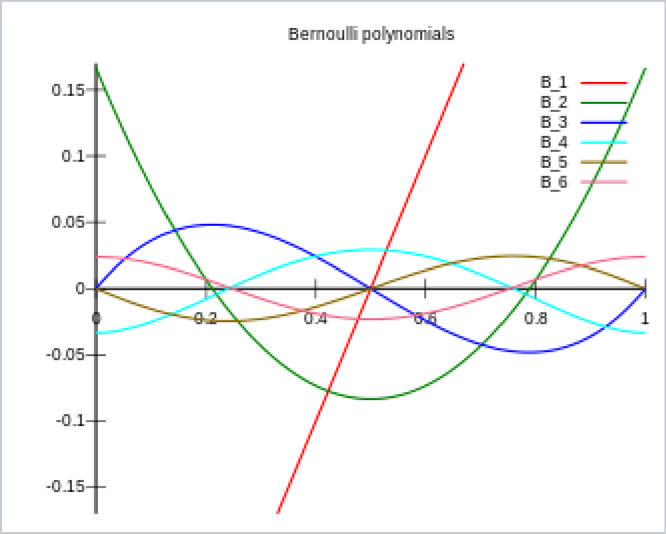
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Bernoulli polynomials

-Jacob Bernoulli

In mathematics, the **Bernoulli polynomials** combine the Bernoulli numbers and binomial coefficients. They are used for series expansion of functions, and with the Euler–MacLaurin formula.

For the Bernoulli polynomials, the number of crossings of the *x*-axis in the unit interval does not go up with the degree. In the limit of large degree, they approach, when appropriately scaled, the sine and cosine functions.



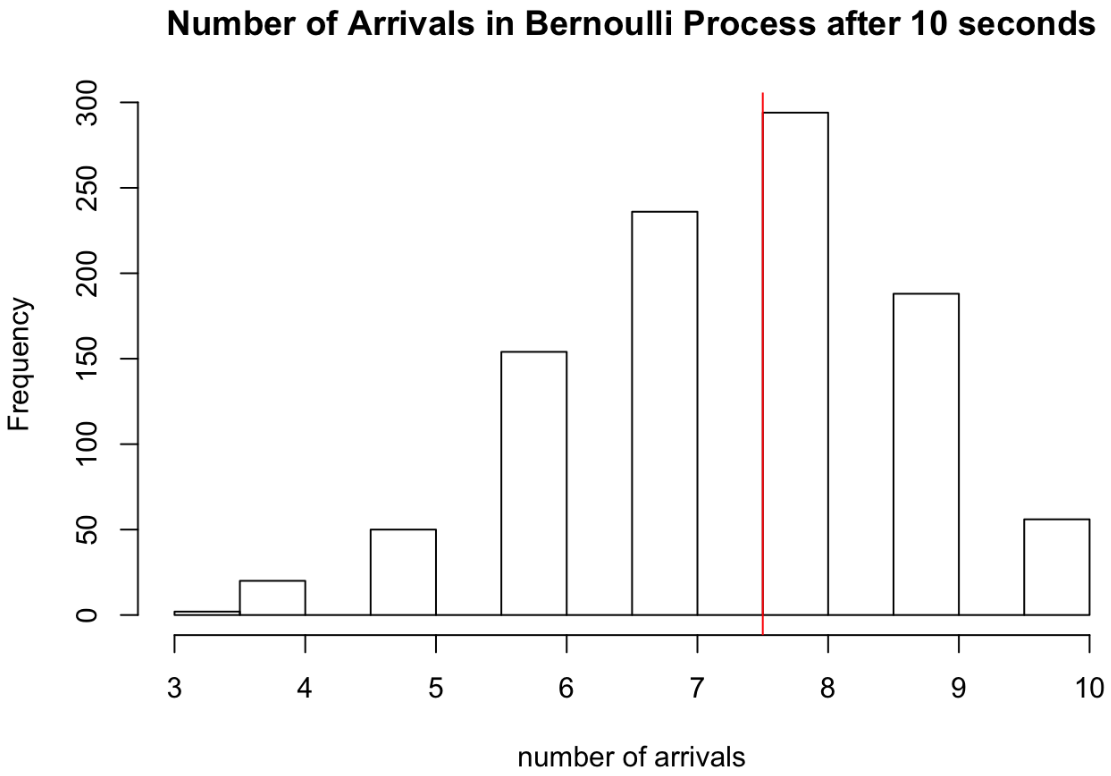
$$B_n(x) = \sum_{s=0}^{n} \binom{n}{s} B_s x^{n-s} \quad (n = 0, 1, \ldots),$$

Bernoulli process

-Jacob Bernoulli

In probability and statistics, a **Bernoulli process** is a finite or infinite sequence of binary random variables, so it is a discrete-time stochastic process that takes only two values, 0 and 1. The component **Bernoulli variables** X_i are identically distributed and independent. A Bernoulli process is a repeated coin flipping, possibly with an unfair coin. Every variable X_i in the sequence is associated with a Bernoulli trial or experiment. They all have the same Bernoulli distribution. Much of what can be said about the Bernoulli process can also be generalized to more than two outcomes (such as the process for a six-sided die); this generalization is known as the Bernoulli scheme.

The problem of determining the process, given only a limited sample of Bernoulli trials, may be called the problem of checking whether a coin is fair.



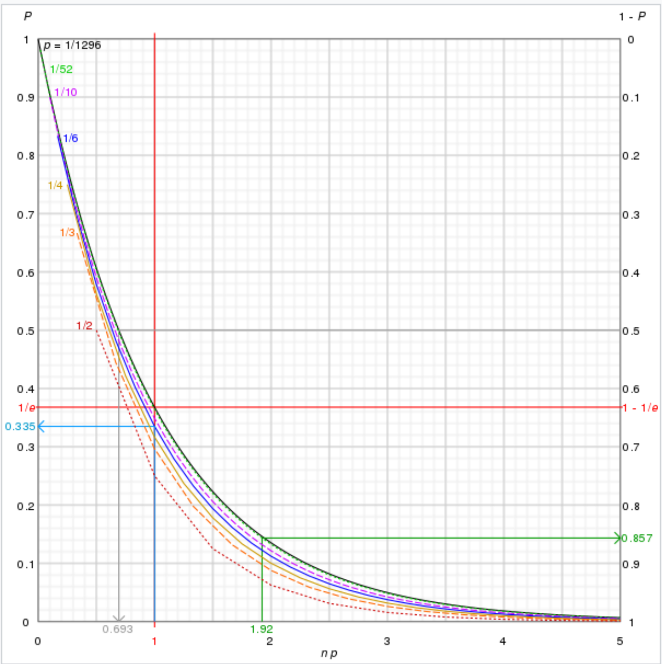
Bernoulli trial

-Jacob Bernoulli

A Bernoulli trial is an experiment that results in two outcomes: *success* and *failure*. One example of a Bernoulli trial is the coin-tossing experiment, which results in heads or tails. In a Bernoulli trial, we define the probability of success and probability of failure as follows:

P[success]=p ,0≤p≤1

P[failure]=1−p



Graphs of probability *P* of **not** observing independent events each of probability *p* after *n* Bernoulli trials vs *np* for various *p*. Three examples are shown:
Blue curve: Throwing a 6-sided die 6 times gives 33.5% chance that 6 (or any other given number) never turns up; it can be observed that as *n* increases, the probability of a 1/*n*-chance event never appearing after *n* tries rapidly converges to *0*.
Grey curve: To get 50-50 chance of throwing a Yahtzee (5 cubic dice all showing the same number) requires 0.69 × 1296 ~ 898 throws.
Green curve: Drawing a card from a deck of playing cards without jokers 100 (1.92 × 52) times with replacement gives 85.7% chance of drawing the ace of spades at least once.

Bernoulli’s triangle

Bernoulli's triangle is an array of partial sums of the binomial coefficients. For any non-negative integer *n* and for any integer *k* included between 0 and *n*, the component in row *n* and column *k* is given by:

$$\sum_{p=0}^{k} \binom{n}{p},$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

11 55 165 330 462 462 330 165 55 11

66 220 495 792 924 792 495 220 66 1