Optimal Impulse Control of West-Nile Virus

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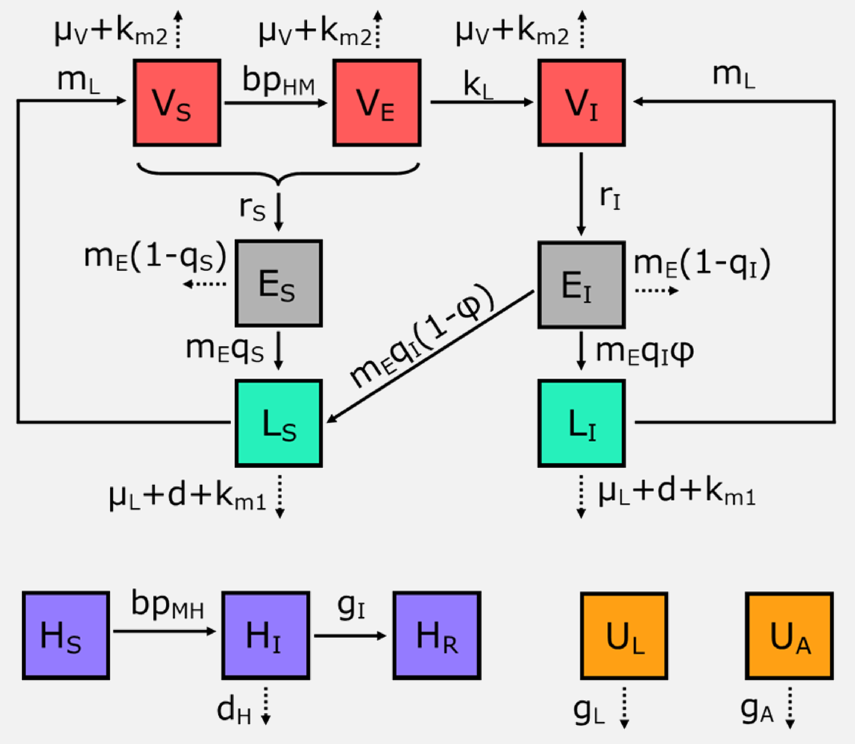
Computational Science

Abstract

We consider a West-Nile Virus transmission model that describes the interaction between bird and mosquito populations (eggs, larvae, adults) and the dynamics for larvicide and adulticide. We derive the basic reproduction number of the infection. We formulate three optimal control problems which seek to balance the cost of insecticide applications (both the timing and application level) with (1) the benefit of reducing the number of mosquitoes, (2) the benefit of reducing the disease burden, or (3) the benefit of preserving the healthy bird population. We derive adjoint equations and establish optimality conditions. Numerical simulations are provided to illustrate the results.

Model Diagram & Variables



Classes: Susceptible (S), Infectious (I), Exposed (E), Recovered (R). Hosts (H) classified as S, I, or R. Eggs (E), larvae (L) classified as S or I. Vectors (V) classified as S, E or I. Larvicide (U_L), adulticide (U_A).

Acknowledgements

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References

[1] Life Cycle of Culex Species Mosquitoes. Center for Disease Control. https://www.cdc.gov/mosquitoes/about/life-cycles/culex.html

[2] M. Canon, C. Cullum and E. Polak. Theory of Optimal Control and Mathematical Programming. McGraw-Hill, 1970.

[3] P. van den Driessche and J. Watmough. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Mathematical Biosciences 180: 29-48. 2002.

[4] S. Lenhart and J.T. Workman. Optimal Control Applied to Biological Models. CRC Press, Boca Raton, 2007.

Culex Pipiens Mosquito Life Cycle

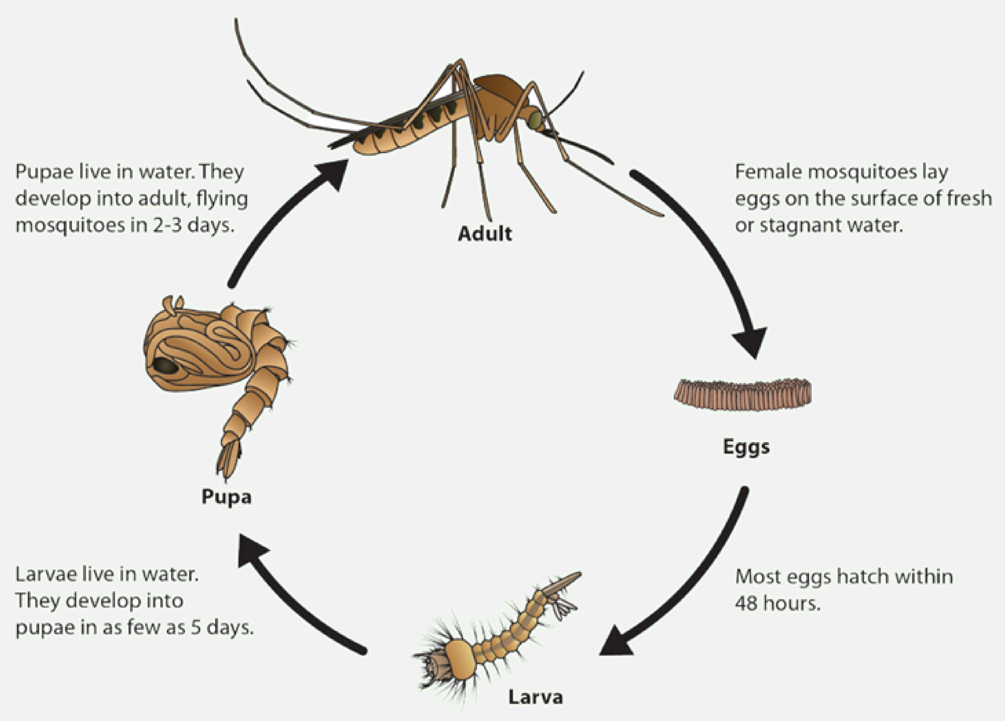


Image sourced from [1]

Our Model

$$\frac{dH_S}{dt} = -p_{MH} b V_I \frac{H_S}{N_H}$$
$$\frac{dH_I}{dt} = p_{MH} b V_I \frac{H_S}{N_H} - d_H H_I - g_I H_I$$
$$\frac{dH_R}{dt} = g_I H_I$$
$$\frac{dE_S}{dt} = r_S(V_S + V_E) - m_E E_S$$
$$\frac{dE_I}{dt} = r_I V_I - m_E E_I$$
$$\frac{dL_S}{dt} = m_E q_S E_S + m_E q_I(1-\phi)E_I - \mu_L L_S - m_L L_S - \frac{d(L_S + L_I)}{C} L_S - k_{m1} L_S U_L$$
$$\frac{dL_I}{dt} = m_E q_I \phi E_I - \mu_L L_I - m_L L_I - \frac{d(L_S + L_I)}{C} L_I - k_{m1} L_I U_L$$
$$\frac{dV_S}{dt} = m_L L_S - \frac{b p_{HM} V_S H_I}{N_H} - \mu_V V_S - k_{m2} V_S U_A$$
$$\frac{dV_E}{dt} = \frac{b p_{HM} V_S H_I}{N_H} - k_L V_E - \mu_V V_E - k_{m2} V_E U_A$$
$$\frac{dV_I}{dt} = m_L L_I + k_L V_E - \mu_V V_I - k_{m2} V_I U_A$$
$$\frac{dU_L}{dt} = -g_L U_L$$
$$\frac{dU_A}{dt} = -g_A U_A$$

R_0 Calculation

The basic reproduction number $\mathscr{R}_0$ is calculated using the next generation matrix method [3].

$$\mathscr{R}_0 = \frac{r_I m_L q_I \phi}{2\mu_V n_2} + \frac{1}{2}\sqrt{\frac{r_I^2 m_L^2 m_E^2 q_I^2 \phi^2}{\mu_V^2 m_S^2 n_2^2} + \frac{4b^2 p_{MH} p_{HM} k_I V_S^*}{\mu_V n_1 n_3 H_S^*}}$$

We estimate that $\mathscr{R}_0$ varies between 0 and 47.88, depending on environmental conditions. We estimate $\mathscr{R}_0 = 1.51$ within a typical residential area.

The Impulse Control Problem

Controls:

$u_L(i)$: normalized amount of larvicide applied at the i^{th} treatment time.

$u_A(i)$: normalized amount of adulticide applied at the i^{th} treatment time.

$\tau(i)$: the i^{th} waiting time.

The waiting times determine the treatment times as follows:

$$T(i) = T(i-1) + \tau(i) = \sum_{k=1}^{i} \tau(k).$$

The control instantaneously adjusts the level of larvicide and adulticide at the treatment times:

$$U_L(T(i)^+) = U_L(T(i)^-) + u_L(i)$$
$$U_A(T(i)^+) = U_A(T(i)^-) + u_A(i).$$

Constraints:

$0 \le u_L(i) \le 1$
$0 \le u_A(i) \le 1$
$1 \le \tau(i) \le T_f.$
$\sum_{i=1}^{N} \tau(i) = T_f,$

where the number of treatments ($N-1$) and the duration of treatment (T_f) are set in advance.

Objective Functionals:

J_1: Vector Control; J_2: Disease Control; J_3: Host Preservation.

Let $N_V(t) = V_S(t) + V_E(t) + V_I(t)$ and

$$Ctrl = c_L \sum_{i=1}^{N} u_L^2(i) + c_A \sum_{i=1}^{N} u_A^2(i) + c_T \sum_{i=1}^{N} \tau^2(i).$$

$$J_1 = c_V \int_0^{T(N)} N_V(t)\, dt + c_E(E_S + E_I)(T(N)) + Ctrl$$
$$J_2 = c_I \int_0^{T(N)} V_I(t) + H_I(t)\, dt + c_E E_I(T(N)) + Ctrl$$
$$J_3 = Ctrl - c_H H_S(T(N))$$

Methods

We reformulate the impulse control problems as nonlinear optimization problems and derive necessary for control characterization [2]. We implement a forward-backward sweep algorithm [4] to numerically solve for controls that satisfy the necessary conditions.

Results

Vector Control:

We simulate the vector control problem (J_1) with control initiated mid-season (Figure 1a) or at the beginning of the season (Figure 1b).

- When the control begins mid-season, the schedule is irregular, with short initial waiting times.
- Both controls taper off in intensity as we approach the end of the control period.
- When control is initiated at the beginning of the season, we can significantly improve results by optimizing the control schedule.

Figure 1a: Vector Control (J_1) over 60 days with 8 treatments from mid-season

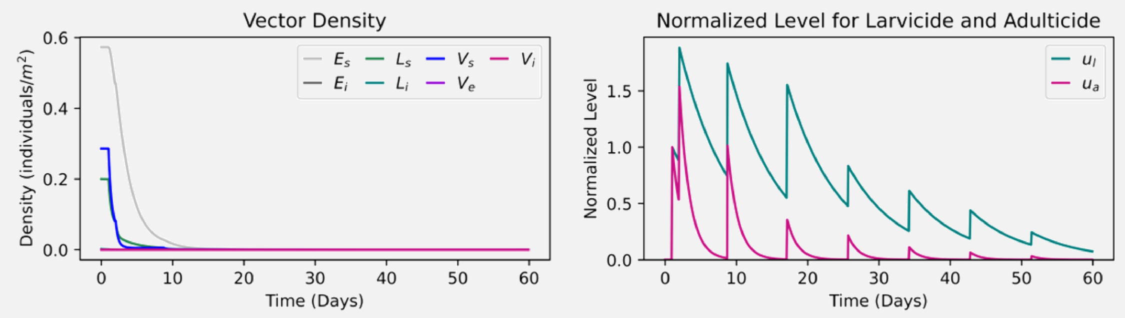
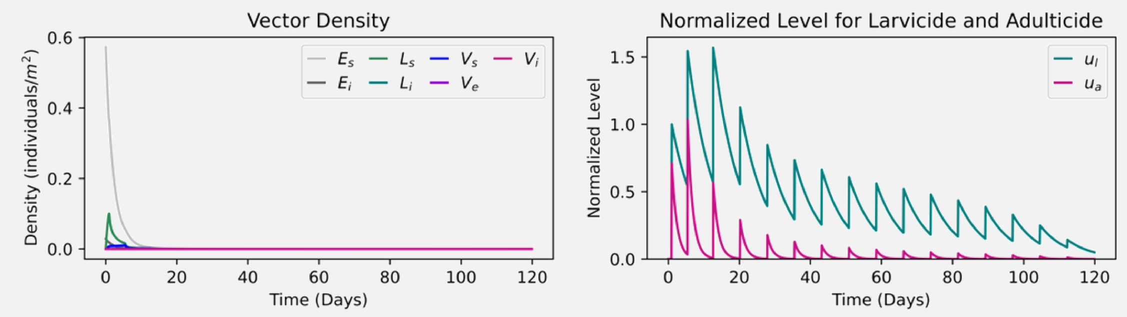


Figure 1b: Vector Control (J_1) over 120 days with 16 treatments from start of season



Disease Control:

We simulate the disease control problem (J_2) with control initiated mid-season (Figure 2a) or at the beginning of the season (Figure 2b).

- In both cases the control schedule is fairly regular and significantly diminishes the vector population.

Figure 2a: Disease Control (J_2) over 60 days with 8 treatments from mid-season

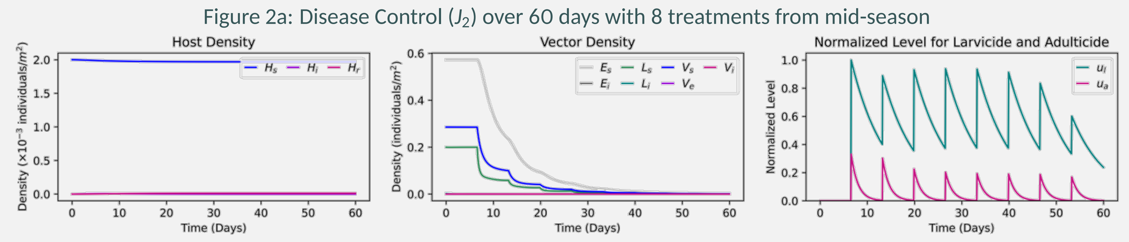
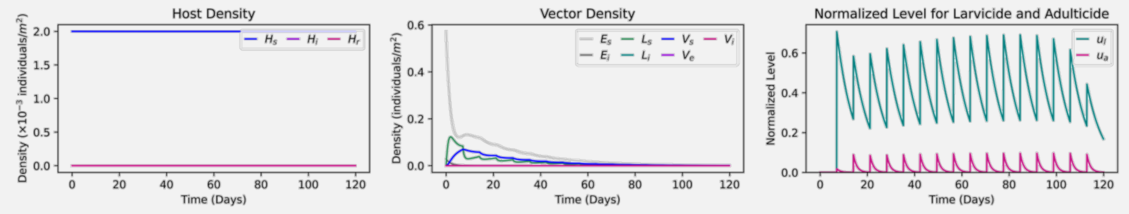


Figure 2b: Disease Control (J_2) over 120 days with 16 treatments from start of season

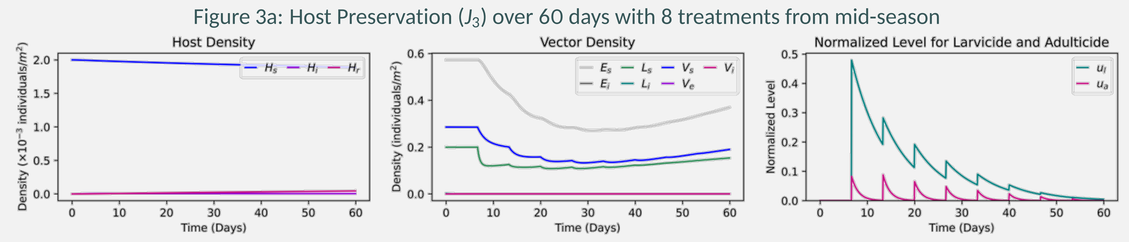


Host Preservation:

We simulate the host preservation control problem (J_3) with control initiated mid-season (Figure 3c).

- Results show that it is possible to suppress the spread of the disease and preserve the host population without eliminating the vector population.

Figure 3a: Host Preservation (J_3) over 60 days with 8 treatments from mid-season



Summary & Future Research

We have implemented a highly flexible numerical framework for the control of West Nile virus. Three objective functionals equipped with numerous balancing parameters (c_A, c_L, c_H, c_I, and c_E) can be adjusted to fit management objectives. Model simulations demonstrate that it is possible to achieve a range of outcomes and that in some situations, it is beneficial to optimize the control schedule. Future work will incorporate bird demographics into the model and explore time-varying environments.